EE 331 – LECT. #7 JAN. 29, 2020

we'll start w/ the TL eqs as phasors:

$$-\frac{dV_s}{dz} = (R + j\omega L)\, I_s \qquad (1.1)$$

$$-\frac{dI_s}{dz} = (G + j\omega C)\, V_s \qquad (1.2)$$

these are coupled first-order ode's. we decouple by taking the deriv of (1.1) wrt z & use the result in (1.2) to get:

$$\frac{d^2 V_s}{dz^2} - (R + j\omega L)(G + j\omega C)\, V_s = 0$$

similarly, we get:

$$\frac{d^2 I_s}{dz^2} - (R + j\omega L)(G + j\omega C)\, I_s = 0$$

now let

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta \; [1/m]$$

then,

$$\boxed{\frac{d^2 V_s}{dz^2} - \gamma^2 V_s = 0}$$

$$\boxed{\frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0}$$

these are wave eqs in phasor form, 2 ode's the solns to the wave eqs

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} = \overset{\text{inc wave}}{V_0^+ e^{-\alpha z - j\beta z}} + \underset{\text{refl wave}}{V_0^- e^{\alpha z + j\beta z}} \quad (2.1)$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} = I_0^+ e^{-\alpha z - j\beta z} + I_0^- e^{\alpha z + j\beta z} \quad (2.2)$$

recall, TL eqs in phasor form:

$$-\frac{dV_s}{dz} = (R + j\omega L) I_s, \quad -\frac{dI_s}{dz} = (G + j\omega C) V_s$$

using (2.1) & (2.2) & the TL eqs, find the relationship btwn V_0^+ & I_0^+ and btwn V_0^- & I_0^-.

$$-\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z} = -(R + j\omega L)(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z})$$

$$\rightarrow \gamma V_0^+ \cancel{e^{-\gamma z}} = (R + j\omega L) I_0^+ \cancel{e^{-\gamma z}} \quad \text{inc wave}$$

$$V_0^+ = \frac{(R + j\omega L)}{\gamma} I_0^+ \rightarrow V_0^+ = (\ ) I_0^+$$

similarly,

$$V_0^- = -\frac{(R + j\omega L)}{\gamma} I_0^-$$

characteristic impedance of the TL:

$$Z_0 = \frac{(R + j\omega L)}{\gamma} \quad [\Omega]$$