EE 331 – LECT. #8 JAN. 31, 2020

on wednesday we ended up with:

characteristic impedance of a TL

$$Z_0 = \frac{R + j\omega L}{\gamma} = \frac{R + j\omega L}{\sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad [\Omega]}$$

recall,

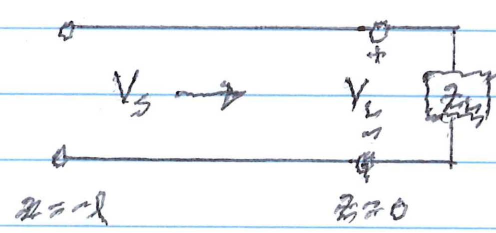
$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad [V]$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

note: γ & Z_0 are known for a given TL & freq

we now have just 2 unknowns: V_0^+ & V_0^-

voltage reflection coefficient



$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$ same for $I_s(z)$

wave traveling toward load; wave travelling away from load (reflected by load) at $z=0$:

$$V_s(0) = V_L = V_0^+ + V_0^-$$

$$I_s(0) = I_L = V_0^+/Z_0 - V_0^-/Z_0$$

but

$$Z_L = \frac{V_L}{I_L} = \frac{V_0^+ + V_0^-}{V_0^+/Z_0 - V_0^-/Z_0} = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}$$

solve for V_0^- using a bunch of algebra:

$$V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+ = \Gamma_L V_0^+$$

$$\rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad [\cdot] \text{ unitless}$$

Γ_L can be complex!

$$\Gamma_L = |\Gamma| e^{j\theta_r} \quad [\cdot]$$

$|\Gamma|$ = magnitude
θ_r = phase

use Γ_L to get:

$$V_s(z) = V_0^+ \left(e^{-\gamma z} + \Gamma_L e^{\gamma z}\right) \quad [V]$$

$$I_s(z) = \frac{V_0^+}{Z_0} \left(e^{-\gamma z} - \Gamma_L e^{\gamma z}\right) \quad [A]$$