EE 331 - LECT. #9 FEB. 3, 2020

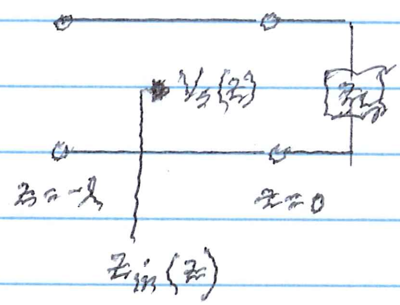
on Friday we found:

$$V_s(z) = V_0^+\left(e^{-\gamma z} + \Gamma_L e^{\gamma z}\right) \; [V] \qquad \Gamma_L = |\Gamma_L| e^{j\theta_r} = \frac{Z_L - Z_0}{Z_L + Z_0} \; [\cdot]$$

$$I_s(z) = \frac{V_0^+}{Z_0}\left(e^{-\gamma z} - \Gamma_L e^{\gamma z}\right) \; [A] \qquad = \text{voltage refl. coeff.}$$

now only 1 unknown - V_0^+

<u>input impedance</u> - differs from book



for now consider Z_{in} at $z=-l$.

$$V_s(-l) = V_0^+\left(e^{\gamma z} + \Gamma_L e^{-\gamma z}\right)$$

$$I_s(-l) = \frac{V_0^+}{Z_0}\left(e^{\gamma z} - \Gamma_L e^{-\gamma z}\right)$$

$$Z_{in} = \frac{V_s(-l)}{I_s(-l)} = Z_0\left(\frac{e^{\gamma z} + \Gamma_L e^{-\gamma z}}{e^{\gamma z} - \Gamma_L e^{-\gamma z}}\right)$$

recall,

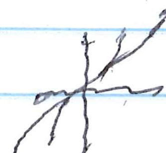
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in} = Z_0 \left(\frac{e^{\gamma \ell} + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-\gamma \ell}}{e^{\gamma \ell} - \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-\gamma \ell}} \right)$$

note that:

$$\frac{e^{x} + e^{-x}}{2} = \cosh x, \quad \frac{e^{x} - e^{-x}}{2} = \sinh x,$$

$$\tanh = \frac{\sinh x}{\cosh x}$$

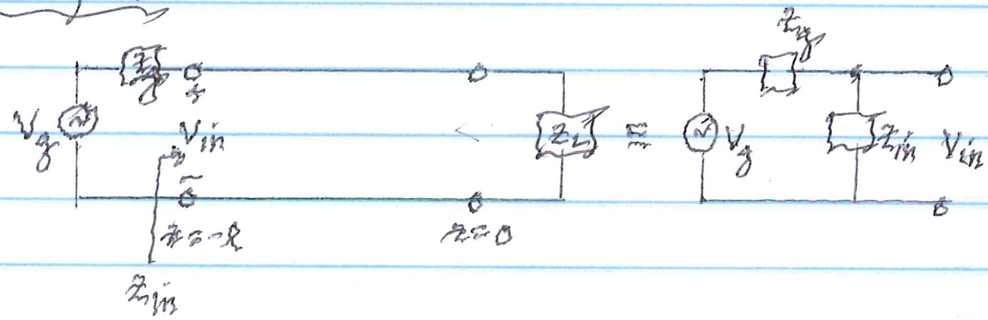


voila (after a lot of algebra):

$$\boxed{Z_{in} = Z_0 \left(\frac{Z_L + Z_0 \tanh(\gamma \ell)}{Z_0 + Z_L \tanh(\gamma \ell)} \right) \qquad (3)}$$

= input impedance at start of TL

<u>finding V_0^+</u> - last unknown



from ckt theory we can find V_{in} in terms of V_g using voltage divider:

$$V_{in} = \left(\frac{Z_{in}}{Z_{in} + Z_g} \right) V_g = V_s(-\ell)$$

$$\left(\frac{Z_{in}}{Z_{in}+Z_g}\right)V_g = V_0^+\left(e^{\gamma l}+\Gamma_L e^{-\gamma l}\right)$$

solve for V_0^+:

$$V_0^+ = \left(\frac{Z_{in}}{Z_{in}+Z_g}\right)\frac{V_g}{\left(e^{\gamma l}+\Gamma_L e^{-\gamma l}\right)} \quad \{V\} = |V_0^+|e^{j\phi}$$

Fig in book