EE 530 – Exam # 3

(April 19, 1999)

Name:

Score:

1(a):

1(b):

1(c):

1(d):

2:

Total:

- This is a "take-home" exam, due by 12:00 noon, Thursday, April 22, 1999.
- You may not consult with any human, except the course-instructor, while solving these problems. Even casual comments like "Gee, wasn't problem 2 easy?" to a class-mate are prohibited.
- You may neither seek nor offer help to any class-mate in solving this exam.
- Exchange of emails, files, programs, course-related material, etc. with class-mates is also prohibited.
- Any violation of these rules will be viewed seriously and will be dealt with according to WSU and EECS policy on academic integrity.
- Show **all** steps to get full credit.
- This exam carries a weight of 20% towards the final grade.
- Unless mentioned otherwise, all random signals are zero-mean wide-sense stationary processes.

1. (**90 points**)

   Consider the following scheme for *adaptive noise cancellation*. Here $x(k) = s(k) + v(k)$ is our observed data which has a *signal* component $s(k)$ and a *noise* component $v(k)$. The signals $s(k)$ and $v(k)$ are uncorrelated. Suppose we are able to measure $w(k)$, which is correlated with the noise $v(k)$ but not with $s(k)$.[1]

   (a) (15 points) First let us assume that we know all the required auto-correlation and cross-correlation functions. Derive the least-squares FIR filter $h(k)$ to minimize $\mathrm{E}(z^2(k))$ where $y(k) = (w * h)(k)$ and $z(k) = x(k) - y(k)$, in terms of the appropriate auto-correlation and cross-correlation functions. Show that minimizing $\mathrm{E}(z^2(k))$ is equivalent to minimizing $\mathrm{E}((z(k) - s(k))^2)$, so that $z(k)$ is the least-squares estimate of the signal $s(k)$.

   (b) (15 points) Now consider the adaptive case, where we do not know the correlation functions but have measurements of $x(k)$ and $w(k)$.[2] For the adaptive filter $h(k)$, $w(k)$ is the "observation data" and $x(k)$ is the "desired signal" and the "error" $z(k) = x(k) - y(k)$ is the (adaptive) estimate of the "signal" $s(k)$. The idea is to adapt the filter $h(.)$ so that its output $y(k)$ is an estimate of the noise $v(k)$, which is then used to "cancel" the noise from our observation $x(k)$. Write the LMS algorithm iterations for this problem (you do not have to derive anything; clearly list all the steps/equations in the LMS algorithm in terms of the various signals in this problem). What is the range of the step size μ.

   (c) (40 points) Download the ASCII files `s.dat`, `w.dat`, and `x.dat`, which contain 500 points of the respective sequences, from the course webpage. Using the LMS algorithm, and the given sequences w, x, obtain an estimate $\hat{s}(k)$ of the signal $s(k)$. Plot the error $s(k) - \hat{s}(k)$. Choose the FIR filter order $n = 2$. You may either use a fixed step size or a variable step-size, as discussed in class, but state what you use, clearly. Do this for three typical values of μ or β.

   (d) (20 points) Repeat the previous part for the RLS algorithm.

2. (**10 points**) In the above problem, suppose the "noise" $v(k)$ is an interference signal, say from a power line. Let $v(k) = A \sin(\frac{2\pi k}{N} + \phi)$, where the amplitude A and phase ϕ are unknown but the integer N (that denotes time period) is known. What would be an appropriate choice for the signal $w(k)$ for "cancelling" this interference signal. Note that this signal $w(k)$ can be deterministic and it is desirable that it be correlated with $v(k)$.

   Note 1: In this case, the interference can be thought of as a (periodic) deterministic signal. For periodic signals $x(k)$, $y(k)$, with period N, the cross-correlation and auto-correlation are defined as $r_{xx}(l) = \frac{1}{N} \sum_k x(k+l)x(k)$, and $r_{xy}(l) = \frac{1}{N} \sum_k x(k+l)y(k)$, where the summations are over one period.

   Note 2: There isn't a whole lot of math involved here. Just some intuition.

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[1] This can typically be done by measuring some ambient noise signal.

[2] Recall that $w(k)$ is uncorrelated with $s(k)$ but is correlated with $v(k)$ (and we do not know the cross-correlation function).