EE 530 – Homework 4

(due March 11, 1999)

1. In this problem, we shall show that the FIR whitening filters are minimum-phase. This will be done in several steps.

   (a) Show that the sequence α_m is non-negative and monotone decreasing, i.e.,

   $$\alpha_0 \geq \alpha_1 \geq \cdots \geq \alpha_m \geq \cdots \geq 0.$$

   (b) Use the above result to show that the reflection coefficients have magnitude at most 1, i.e. $|c_m| \leq 1$ for every integer m.

   (c) In this part, we will assume that $|c_m| < 1$ and show that the zeros of the FIR whitening filter $A_m(z)$ all lie strictly inside the unit circle.[1] We will use induction on the filter order and the Levinson-Durbin recursion.

   - First show that the zero of $A_1(z)$ lies inside the unit circle. You may need the result of Problem 7.15 here.

   Now assume that the zeros of $A_{m-1}(z)$ are all (strictly) inside the unit circle and let z_i be a zero of $A_m(z)$ i.e., $A_m(z_i) = 0$.

   - Using the Levinson-Durbin recursion, show that

   $$\frac{1}{c_m} = -\frac{z_i^{-m} A_{m-1}(z_i^{-1})}{A_{m-1}(z_i)} = Q(z_i) \quad \text{(say)}.$$

   Note that $P(z) = -zQ(z)$ corresponds to the transfer function of an all pass system. Hence, we can write

   $$P(z) = -zQ(z) = \prod_{k=1}^{m-1} \frac{1 + zw_k}{z + w_k},$$

   where w_k are the roots of $A_{m-1}(z)$. Therefore, by the induction hypothesis $|w_k| < 1$. Observe that $|z| < 1 \Rightarrow |P(z)| > 1$, $|z| = 1 \Rightarrow |P(z)| = 1$, and $|z| > 1 \Rightarrow |P(z)| < 1$.

   - Now argue that $|Q(z_i)| = \frac{1}{|c_m|} > 1$ implies that $|z_i| < 1$.

   This completes the induction step and shows that $A_m(z)$ is minimum phase.

2. Problem 7.12 from text.

3. Problem 7.15 from text.

4. Problem 7.21 from text.

The following problems are for practice. You are urged to work them out but you don't have to turn them in. See Chapter 11 (specially section 11.4) in Proakis for more examples. The "stochastic version" of the whitening problem is considered there. It may take a while to get used to their notation though!

1. Suppose $a_m(n)$ is an order m FIR whitening filter and $f_m(n) = (g * a_m)(n)$ is the forward prediction error sequence. Let $\tilde{a}_m(n)$ be the corresponding backward prediction filter ($\tilde{a}_m(n) = a_m(m-n)$, $n = 0, 1, \ldots, m$) and define $b_m(n) = (g * \tilde{a}_m)(n)$ to be the backward prediction error sequence.

[1] If $|c_m| = 1$, $A_m(z)$ may have some zeros on the unit circle.

(a) Show that

$$\sum_n f_m(n) f_l(n) = \alpha_{\max(m,l)}.$$

(b) Show that the backward prediction error sequences are orthogonal in the following sense:

$$\sum_n b_m(n) b_l(n) = \begin{cases} 0 & m \neq l \\ \alpha_m & m = l. \end{cases}$$

2. Problems 7.13, 7.16, 7.22 from text.