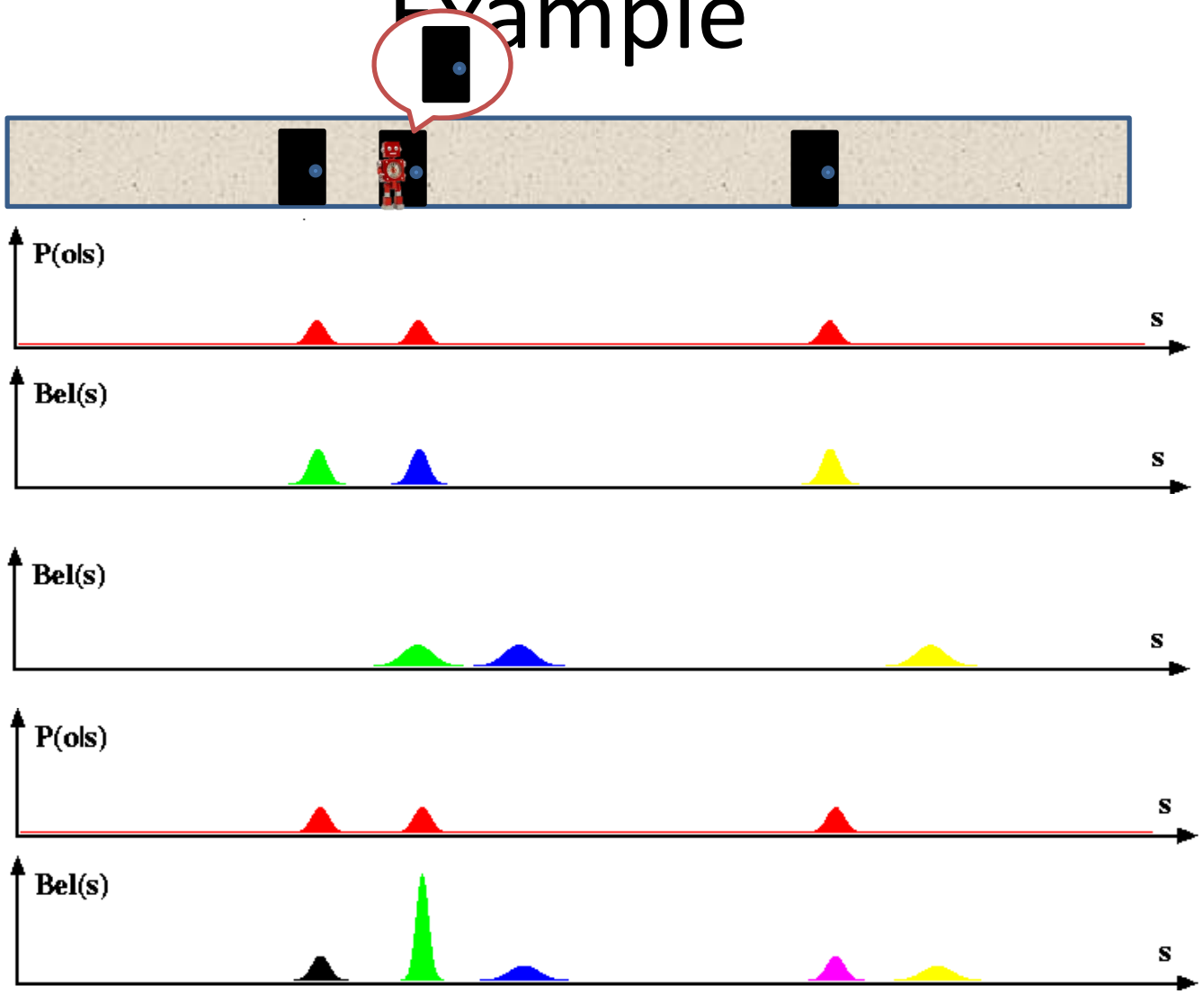
- SLAM exercise?

- Google Cartographer

https://opensource.googleblog.com/2016/10/introducing-cartographer.html?m=1

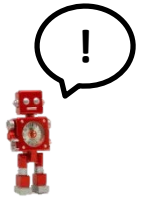
Example



Example 2

0.04	0.12	0.12	0.04	0.04

Sums to 0.36



Normalize (divide by total)

The probability distribution no longer sums to 1!

.111	.333	.333	.111	.111

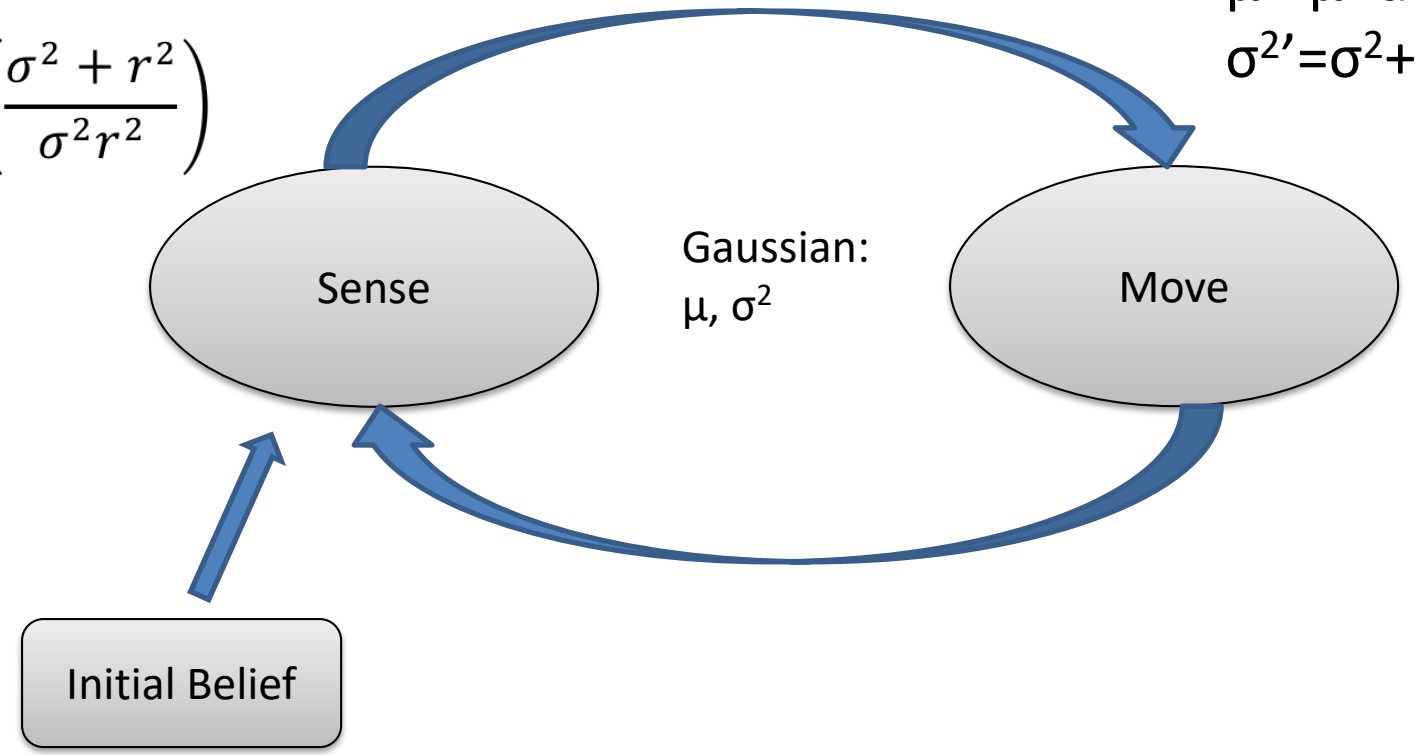
Kalman Filter

$$\mu' = \left(\frac{xr^2 + v\sigma^2}{\sigma^2 + r^2}\right)$$

$$\sigma^{2\prime} = \left(\frac{\sigma^2 + r^2}{\sigma^2 r^2}\right)$$

$\mu' = \mu + u$

$\sigma^{2\prime} = \sigma^2 + r^2$



Implementing a Kalman Filter example

VERY simple model of robot movement:

$$x' = x + \dot{x}$$
$$\dot{x}' = \dot{x}$$

$$\begin{bmatrix} x' \\ \dot{x}' \end{bmatrix} \leftarrow [?] \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

What information do our sensors give us? $\longrightarrow$ $Z \leftarrow [?] \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

Implementing a Kalman Filter example

$$x' = x + \dot{x}$$
$$\dot{x}' = \dot{x}$$

$$\begin{bmatrix} x' \\ \dot{x}' \end{bmatrix} \leftarrow \overset{\mathbf{F}}{[?]} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$
$$Z \leftarrow \underset{\mathbf{H}}{[?]} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

H = [1 0]

Implementing a Kalman Filter

Motion Prediction

$$x' = Fx + u$$
$$P' = FPF^T + Q$$

Estimate

- P’: uncertainty covariance
- F: state transition matrix
- u: motion vector
- F: motion noise

Implementing a Kalman Filter

Motion Prediction

$$x' = Fx + u$$
$$P' = FPF^T + Q$$

Measurement Update

$$y = z - Hx$$
$$S = HPH^T + R$$
$$K = PH^T S^{-1}$$

Estimate

- P': uncertainty covariance
- F: state transition matrix
- u: motion vector
- F: motion noise

Implementing a Kalman Filter

Motion Prediction

$$x' = Fx + u$$
$$P' = FPF^T + Q$$

Measurement Update

$$y = z - Hx$$
$$S = HPH^T + R$$
$$K = PH^T S^{-1}$$

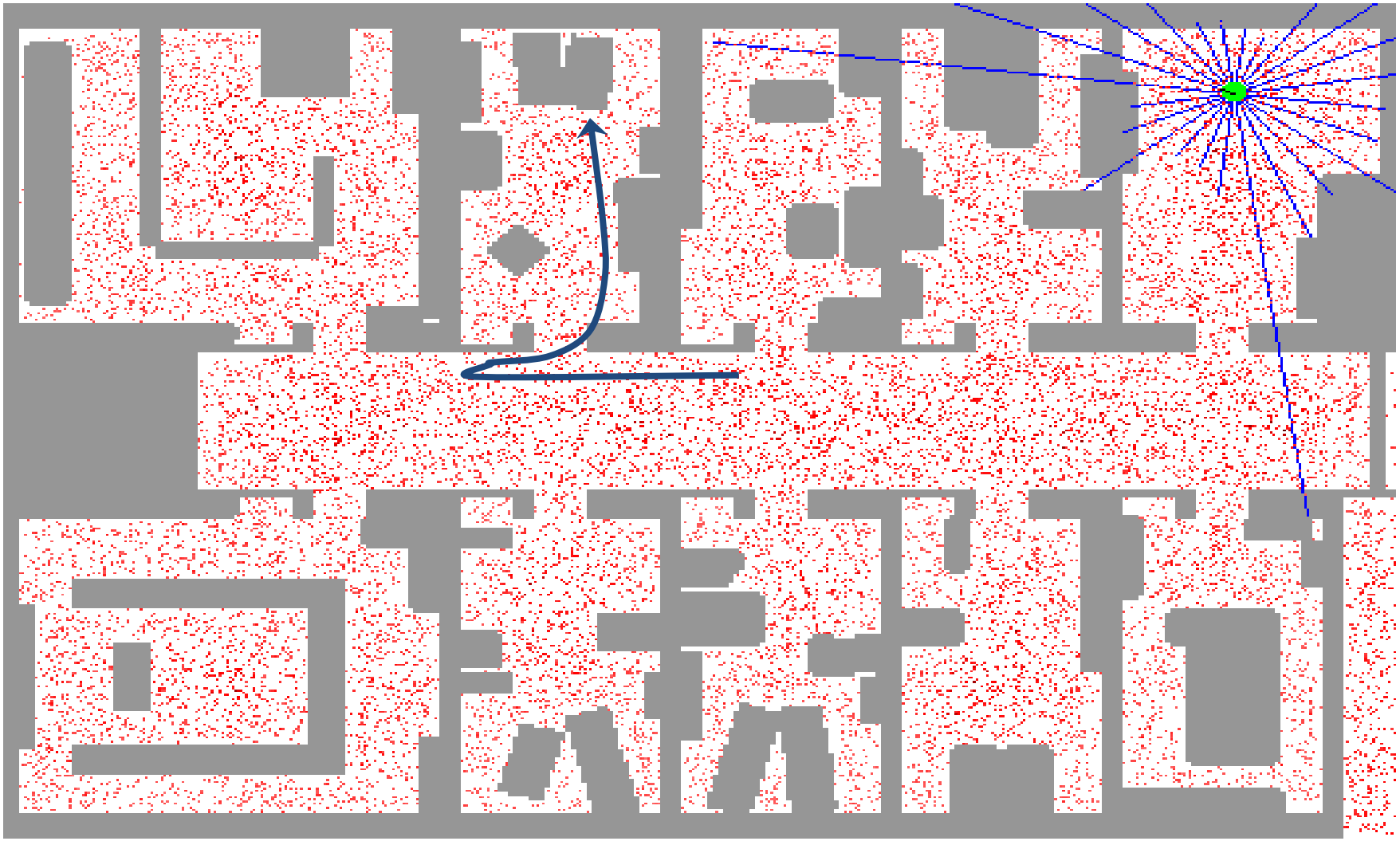
End Result

$$x' = x + (Ky)$$
$$P' = (I - KH)P$$

Estimate

- P’: uncertainty covariance
- F: state transition matrix
- u: motion vector
- F: motion noise

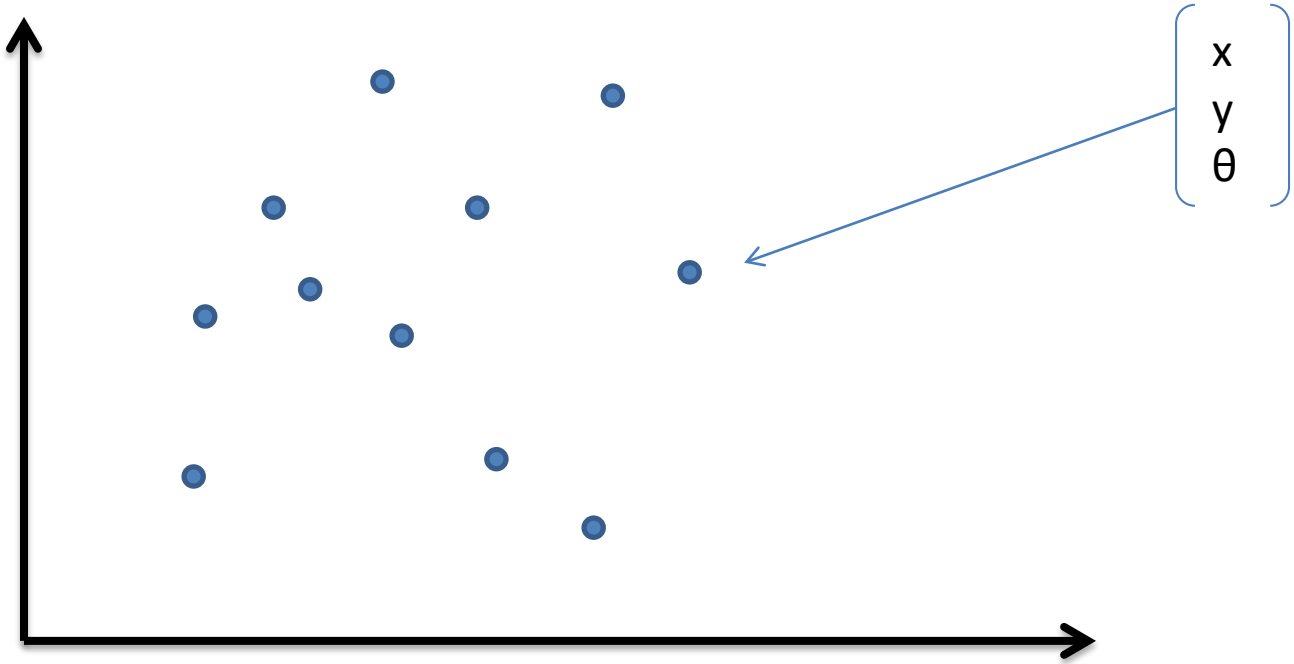
Particle Filter Localization (using sonar)



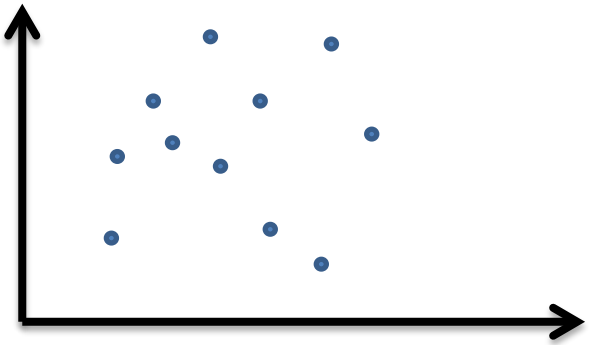
http://www.cs.washington.edu/ai/Mobile_Robotics//mcl/animations/global-floor-start.gif

Particles

- Each particle is a guess about where the robot might be

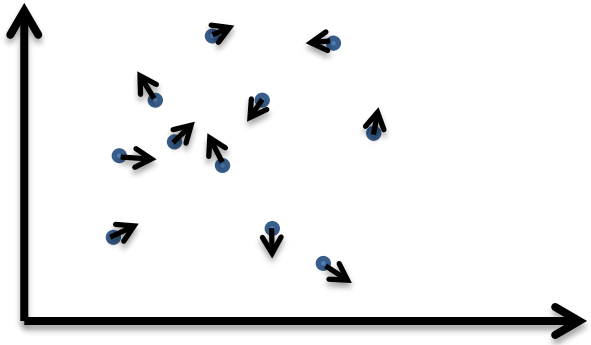


Robot Motion



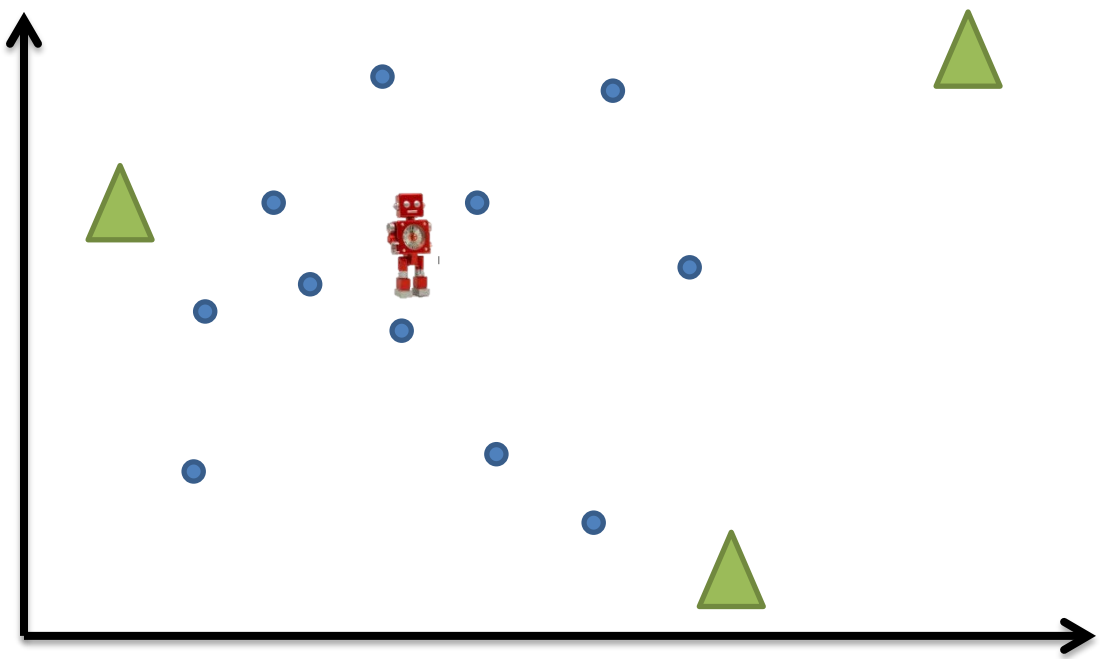
n particles

move each particle
according to the rules of
motion
+
add random noise

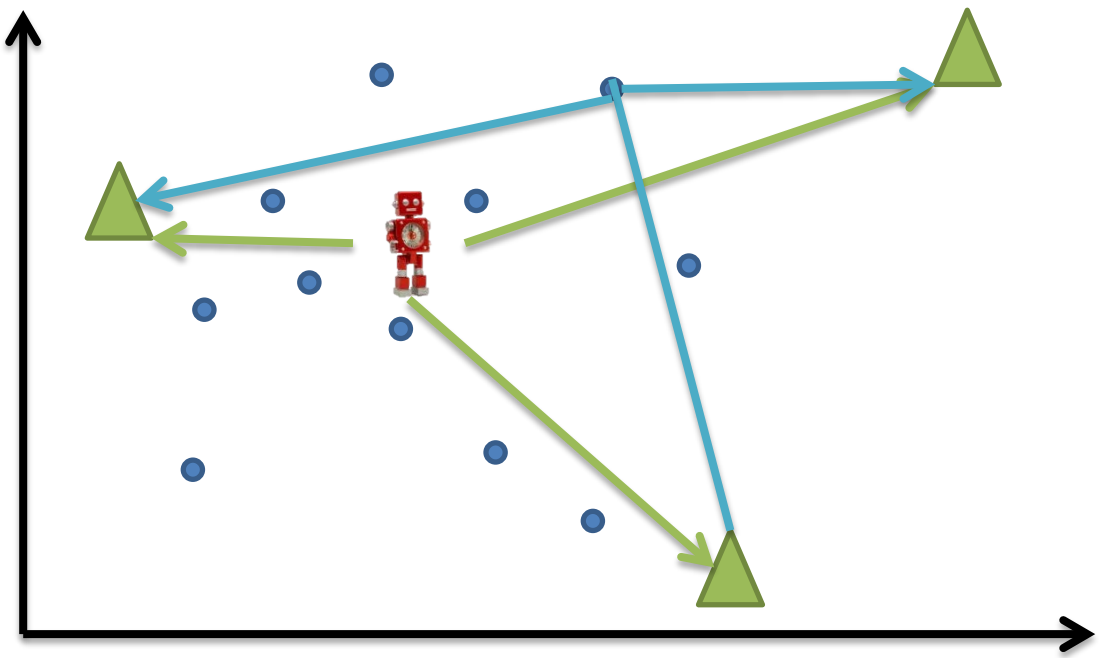


n particles

Incorporating Sensing



Incorporating Sensing



Difference between the
actual measurement
and the
estimated measurement

Importance weight

Importance weight python code

```
def Gaussian(self, mu, sigma, x):

        # calculates the probability of x for 1-dim Gaussian with mean mu and var. sigma
        return exp(- ((mu - x) ** 2) / (sigma ** 2) / 2.0) / sqrt(2.0 * pi * (sigma ** 2))


def measurement_prob(self, measurement):

        # calculates how likely a measurement should be
        prob = 1.0;
        for i in range(len(landmarks)):
            dist = sqrt((self.x - landmarks[i][0]) ** 2 + (self.y - landmarks[i][1]) ** 2)
            prob *= self.Gaussian(dist, self.sense_noise, measurement[i])
        return prob


def get_weights(self):
        w = []
        for i in range(N): #for each particle
                w.append(p[i].measurement_prob(Z)) #set it’s weight to p(measurement Z)
        return w
```

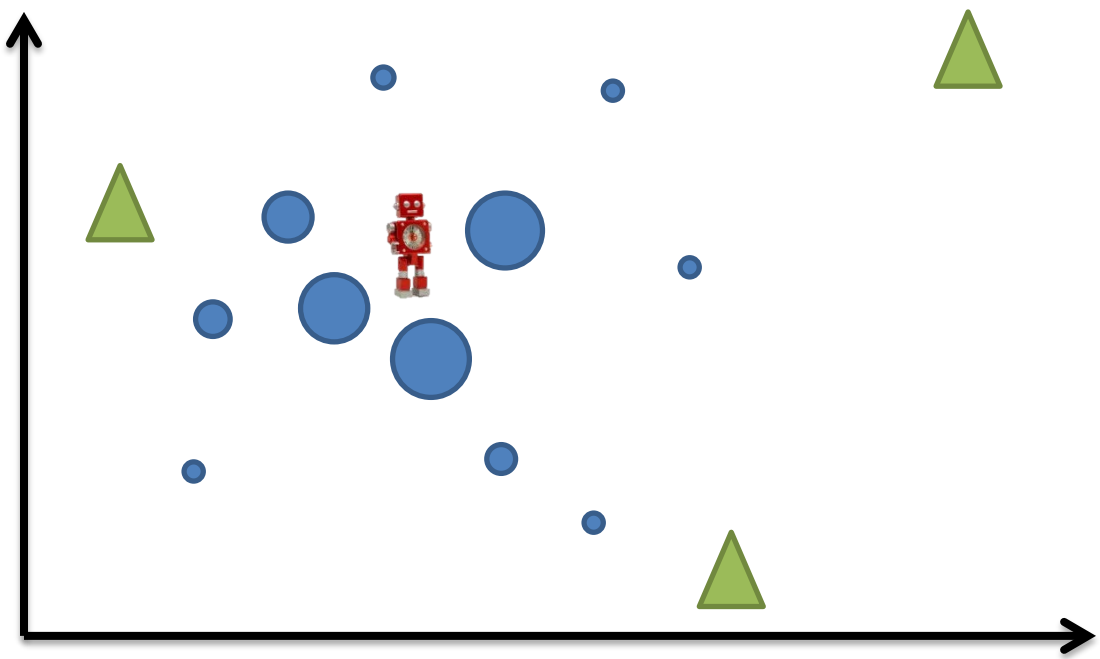
Importance weight pseudocode

Calculate the probability of a sensor measurement for a particle:

```
prob = 1.0;
    for each landmark
        d = Euclidean distance to landmark
        prob *= Gaussian probability of obtaining a reading at
                    distance d for this landmark from this particle

    return prob
```

Incorporating Sensing



Resampling

n original particles	Importance Weight
	0.2
	0.6
	0.2
	0.8
	0.8
	0.2

Sum is 2.8

Resampling

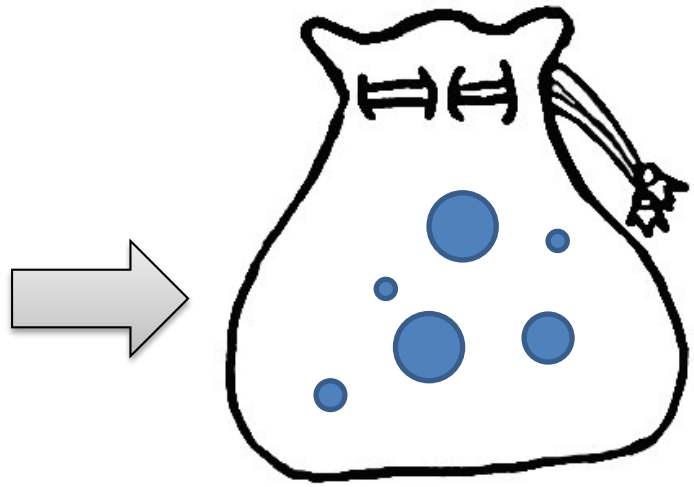
n original particles	Importance Weight	Normalized Probability
	0.2	0.07
	0.6	0.21
	0.2	0.07
	0.8	0.29
	0.8	0.29
	0.2	0.07

Sum is 2.8

Resampling

n original particles	Importance Weight	Normalized Probability
	0.2	0.07
	0.6	0.21
	0.2	0.07
	0.8	0.29
	0.8	0.29
	0.2	0.07

Sum is 2.8



Sample *n* new particles from previous set
Each particle chosen with probability p, with replacement

Res

Is it possible that one of the particles is never chosen?

Yes!

Is it possible that one of the particles is chosen more than once?

Yes!

n original particles	Importance Weight	Normalized Probability
	0.2	0.07
	0.6	0.21
	0.2	0.07
	0.8	0.29
	0.8	0.29
	0.2	0.07

Sum is 2.8

Sample *n* new particles from previous set

Each particle chosen with probability p, with replacement

Res

What is the probability that this particle is not chosen during the resampling of the six new particles?

n original particles	Importance Weight	Normalized Probability
	0.2	0.07
	0.6	0.21
	0.2	0.07
	0.8	0.29
	0.8	0.29
	0.2	0.07

Sum is 2.8

Sample *n* new particles from previous set
Each particle chosen with probability p, with replacement

Question

- What happens if there are no particles near the correct robot location?

Question

- What happens if there are no particles near the correct robot location?

- Possible solutions:
  - Add random points each cycle
  - Add random points each cycle, where is proportional to the average measurement error
  - Add points each cycle, located in states with a high likelihood for the current observations

Summary

- Kalman Filter
  - Continuous
  - Unimodal
  - Harder to implement
  - More efficient
  - Requires a good starting guess of robot location
- Particle Filter
  - Continuous
  - Multimodal
  - Easier to implement
  - Less efficient
  - Does not require an accurate prior estimate

SLAM

- Simultaneous localization and mapping:

  Is it possible for a mobile robot to be placed at an unknown location in an unknown environment and for the robot to incrementally build a consistent map of this environment while simultaneously determining its location within this map?

http://flic.kr/p/9jdHrL

Three Basic Steps

- The robot moves
  - increases the uncertainty on robot pose
  - need a mathematical model for the motion
  - called *motion model*

Three Basic Steps

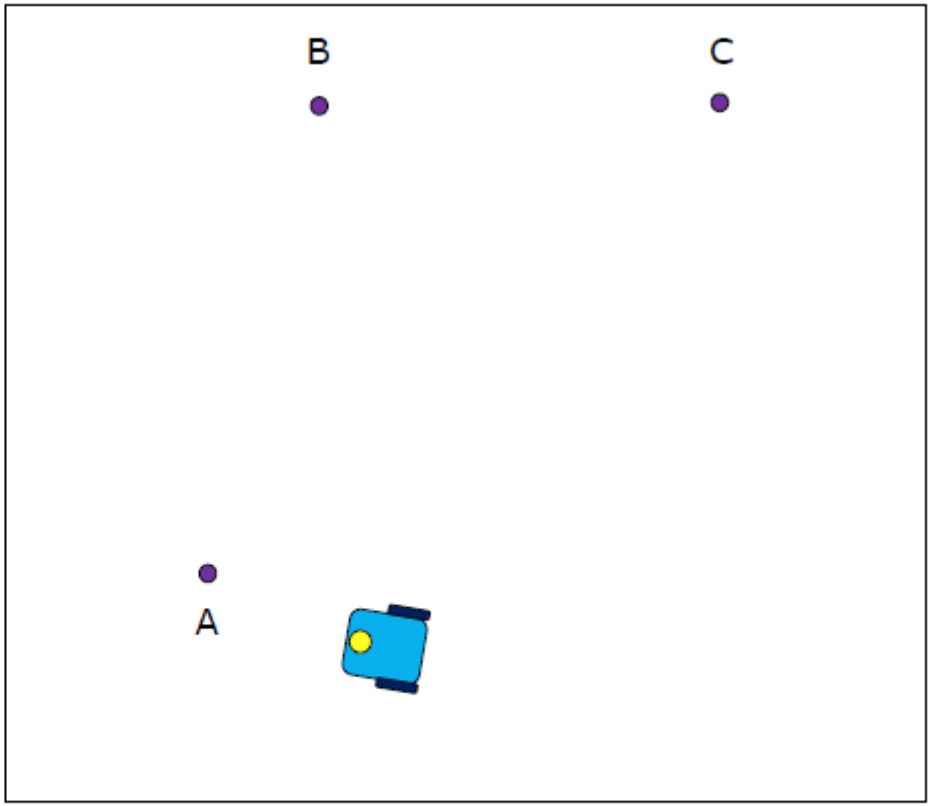
- The robot discovers interesting features in the environment
  - called *landmarks*
  - uncertainty in the location of landmarks
  - need a mathematical model to determine the position of the landmarks from sensor data
  - called *inverse observation model*

Three Basic Steps

- The robot observes previously mapped landmarks
  - uses them to correct both self localization and the localization of all landmarks in space
  - uncertainties decrease
  - need a model to predict the measurement from predicted landmark location and robot localization
  - called *direct observation model*

How to do SLAM

- Use internal representations for
  - the positions of landmarks (: map)
  - the camera parameters
- Assumption: Robot's uncertainty at starting position is zero

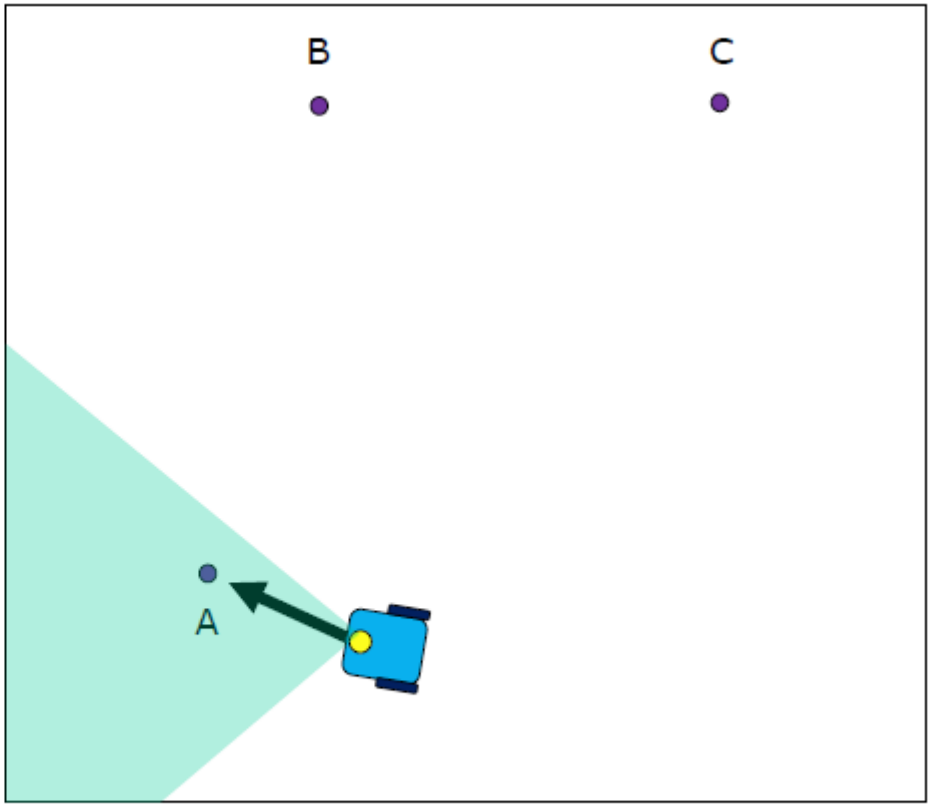


Start: robot has zero uncertainty

How to do SLAM

On every frame:

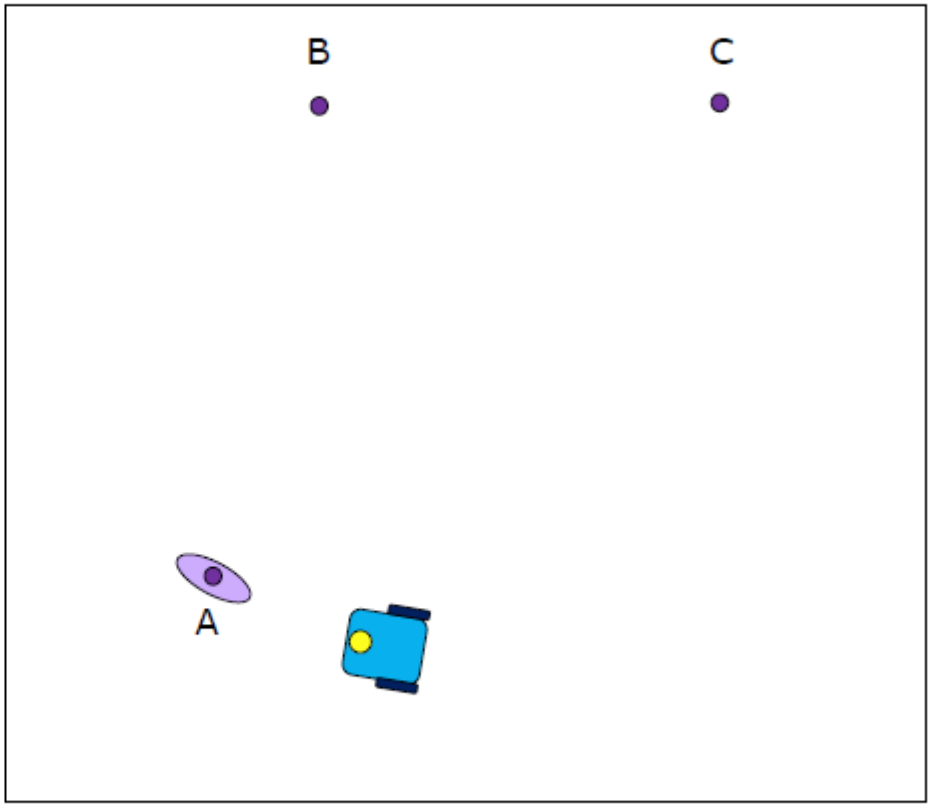
- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



First measurement of feature A

How to do SLAM

- The robot observes a feature which is mapped with an uncertainty related to the **measurement model**



On every frame:

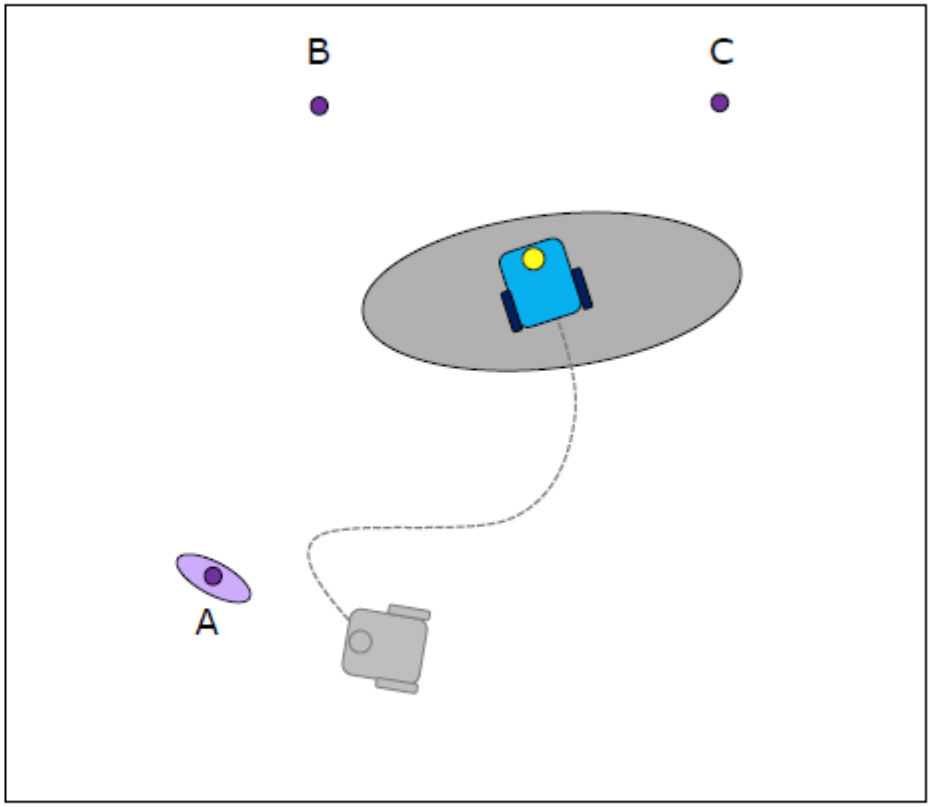
- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations

How to do SLAM

- As the robot moves, its pose uncertainty increases, obeying the robot's **motion model**.

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



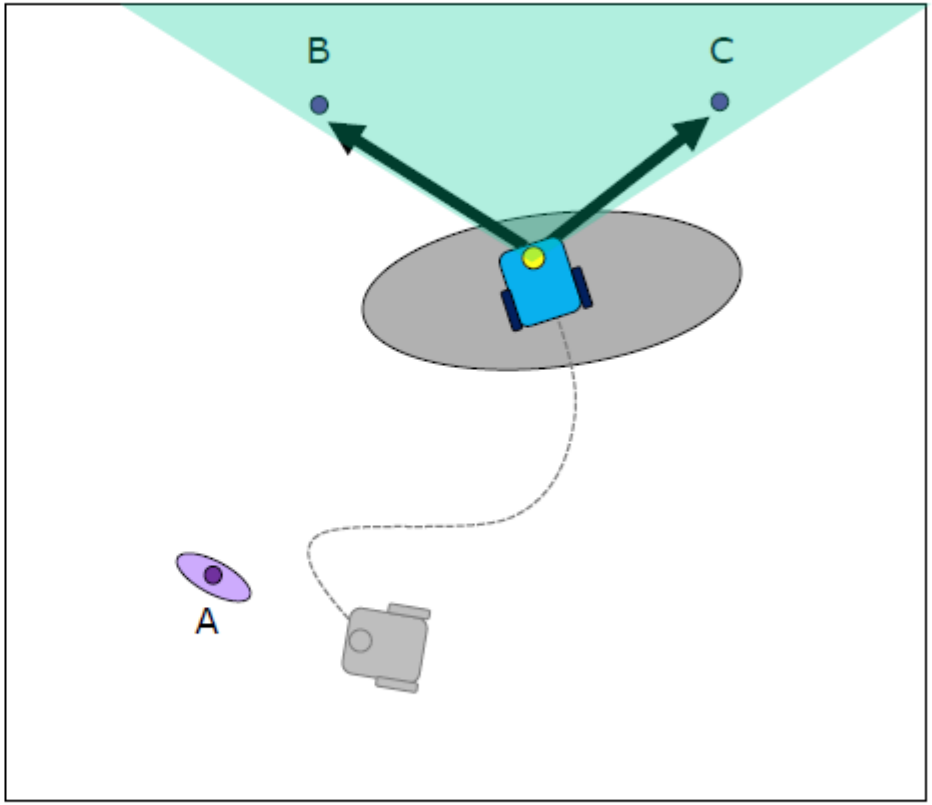
Robot moves forwards: uncertainty grows

How to do SLAM

- Robot observes two new features.

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



Robot makes first measurements of B & C

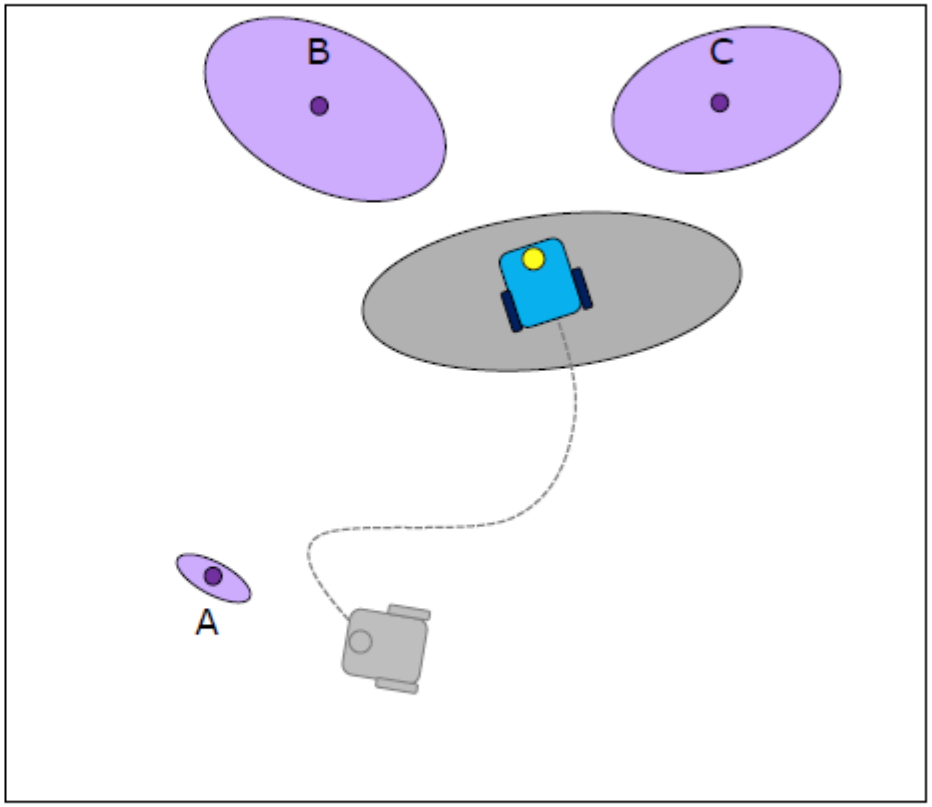
How to do SLAM

- Their position uncertainty results from the combination of the measurement error with the robot pose uncertainty.

⇨ map becomes correlated with the robot pose estimate.

On every frame:

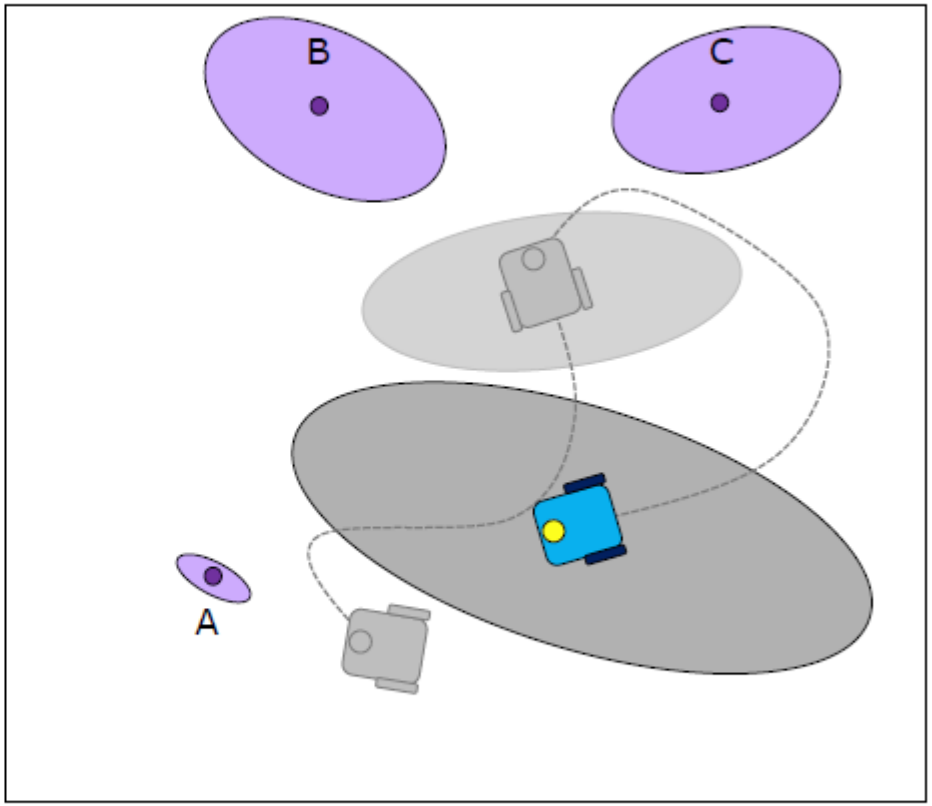
- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



Robot makes first measurements of B & C

How to do SLAM

- Robot moves again and its uncertainty increases (motion model)



Robot moves again: uncertainty grows more

On every frame:

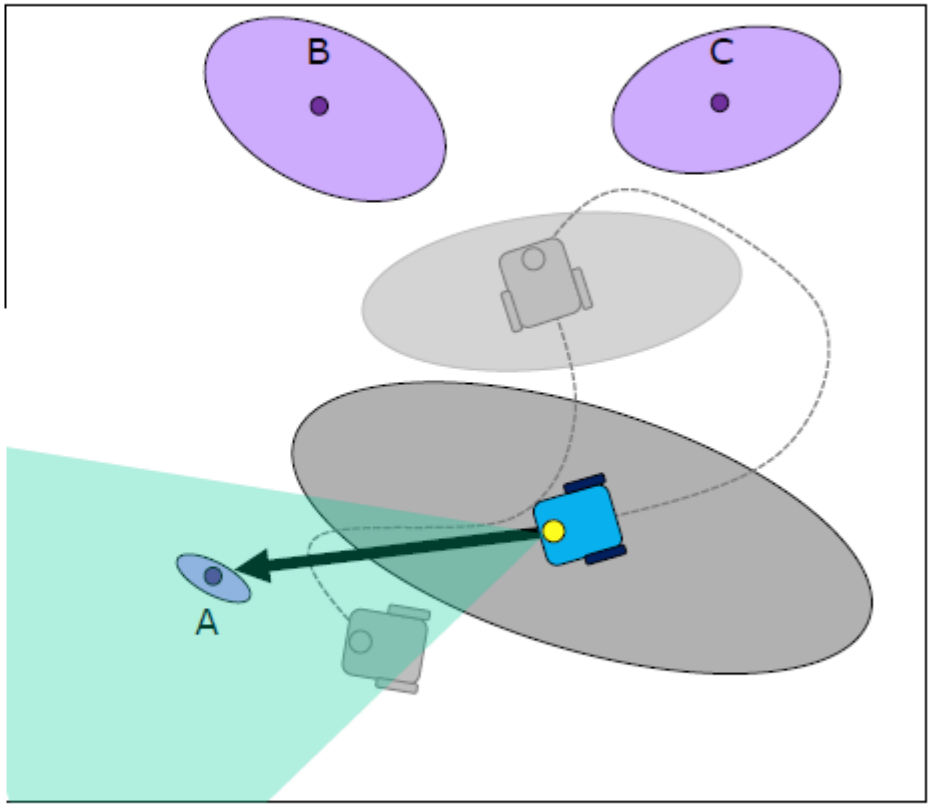
- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations

How to do SLAM

- Robot re-observes an old feature
  ⇨ **Loop closure** detection

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



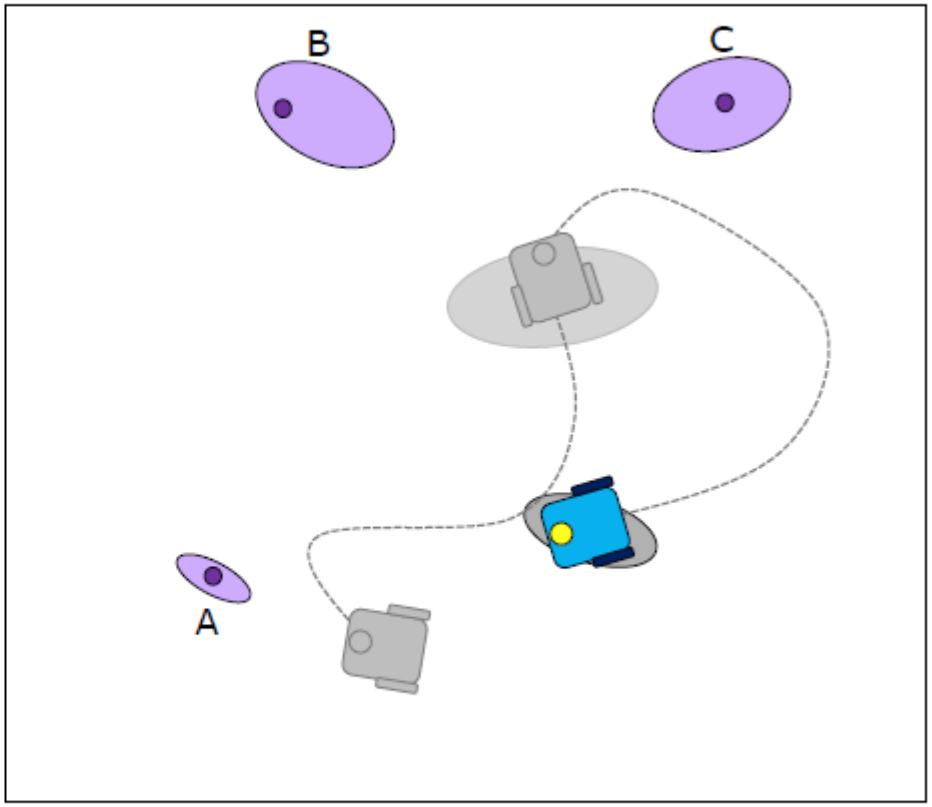
Robot re-measures A: "loop closure"

How to do SLAM

- Robot updates its position: the resulting position estimate becomes correlated with the feature location estimates.
- Robot's uncertainty shrinks and so does the uncertainty in the rest of the map

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



Robot re-measures A: "loop closure"
uncertainty shrinks