• Monday: Lab due
• Lab?
Consider a “block” in a cache

• **Temporal locality:**
  - Recently referenced items are likely to be referenced again in the near future

• **Spatial locality:**
  - Items with nearby addresses tend to be referenced close together in time
2-Level Hierarchy

• A two level hierarchy suggests that memory can be structured in two layers, or two interacting layers.
  – **Primary level**  -- small and fast
  – **Secondary level**  -- large and slow
• This relationship indicates a trade-off in memory technologies between processing time expense and hardware purchasing expense.
Memory Mountain Main Routine

```c
/* mountain.c - Generate the memory mountain. */
#define MINBYTES (1 << 10) /* Working set size ranges from 1 KB */
#define MAXBYTES (1 << 23) /* ... up to 8 MB */
#define MAXSTRIDE 16 /* Strides range from 1 to 16 */
#define MAXELEMS MAXBYTES/sizeof(int)

int data[MAXELEMS]; /* The array we'll be traversing */

int main()
{
    int size; /* Working set size (in bytes) */
    int stride; /* Stride (in array elements) */
    double Mhz; /* Clock frequency */

    init_data(data, MAXELEMS); /* Initialize each element in data to 1 */
    Mhz = mhz(0); /* Estimate the clock frequency */
    for (size = MAXBYTES; size >= MINBYTES; size >>= 1) {
        for (stride = 1; stride <= MAXSTRIDE; stride++)
            printf("%.1f\t", run(size, stride, Mhz));
        printf("\n");
    }
    exit(0);
}
```
Memory Mountain Test Function

/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz) {
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride); /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0); /* call test(elems,stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
The Memory Mountain

Pentium III Xeon
550 MHz
16 KB on-chip L1 d-cache
16 KB on-chip L1 i-cache
512 KB off-chip unified
L2 cache

Slopes of
Spatial
Locality

Ridges of
Temporal
Locality

read throughput (MB/s)

stride (words)

working set size (bytes)

L1

L2

mem
Hits vs misses

\[ t_a = h t_p + (1 - h) t_s \]

- \( t_a \): actual access time
- \( t_p \): primary memory access time
- \( t_s \): secondary memory access time
- \( h \): hit ratio, or hits per memory reference
Hits vs misses

\[ t_a = h t_p + (1 - h) t_s \]

- Assume:
  - \( t_p \) -- primary memory access time is 10 ns.
  - \( t_s \) -- secondary memory access time is 100 ns.
## Hits vs misses

\[ t_a = h t_p + (1 - h) t_s \]

<table>
<thead>
<tr>
<th>h</th>
<th>h t_p</th>
<th>(1-h)t_s</th>
<th>t_a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.2</td>
<td>2</td>
<td>80</td>
<td>82</td>
</tr>
<tr>
<td>0.4</td>
<td>4</td>
<td>60</td>
<td>64</td>
</tr>
<tr>
<td>0.5</td>
<td>5</td>
<td>50</td>
<td>55</td>
</tr>
<tr>
<td>0.6</td>
<td>6</td>
<td>40</td>
<td>46</td>
</tr>
<tr>
<td>0.8</td>
<td>8</td>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td>1.0</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>
Matrix Multiplication Example

• **Major Cache Effects to Consider**
  • Total cache size
    • Exploit temporal locality and keep the working set small (e.g., by using blocking)
  • Block size
    • Exploit spatial locality

• **Description:**
  • Multiply N x N matrices
  • O(N^3) total operations
  • Accesses
    • N reads per source element
    • N values summed per destination
      • but may be able to hold in register

```c
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

Variable `sum` held in register
Miss Rate Analysis for Matrix Multiply

- **Assume:**
  - Cache Line (Block) size = 16 bytes (fits 4 32-bit words)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows

- **Analysis Method:**
  - Look at access pattern of inner loop
Layout of C Arrays in Memory (review)

- **C arrays allocated in row-major order**
  - each row in contiguous memory locations

- **Stepping through columns in one row:**
  - `for (i = 0; i < N; i++)`
    - `sum += a[0][i];`
  - accesses successive elements
  - if block size (B) > 4 bytes, exploit spatial locality
    - compulsory miss rate = 4 bytes / B

- **Stepping through rows in one column:**
  - `for (i = 0; i < n; i++)`
    - `sum += a[i][0];`
  - accesses distant elements
  - no spatial locality!
    - compulsory miss rate = 1 (i.e. 100%)
Matrix Multiplication (ijk)

```c
/* ijk */
for (i=0; i<n; i++)  {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

Misses per Inner Loop Iteration:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.25</td>
<td>1.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Matrix Multiplication (jik)

```c
/* jik */
for (j=0; j<n; j++) {
    for (i=0; i<n; i++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum
    }
}
```

Misses per Inner Loop Iteration:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.25</td>
<td>1.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Matrix Multiplication (kij)

```c
/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

**Misses per Inner Loop Iteration:**
- A
- B
- C

Approx speedup relative to 1.25?

Cache = 10ms and memory = 100ms

n is BIG

\[ t_a = h t_p + (1 - h) t_s \]
Matrix Multiplication (kij)

```c
/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

Inner loop:
- Fixed
- Row-wise
- Row-wise

Misses per Inner Loop Iteration:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Matrix Multiplication (ikj)

/* ikj */
for (i=0; i<n; i++) {
    for (k=0; k<n; k++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}

Misses per Inner Loop Iteration:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Matrix Multiplication (jki)

```c
/* jki */
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

**Misses per Inner Loop Iteration:**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Matrix Multiplication (kji)

/* kji */
for (k=0; k<n; k++) {
    for (j=0; j<n; j++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}

Misses per Inner Loop Iteration:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Summary of Matrix Multiplication

**ijk (& jik):**
- 2 loads, 0 stores
- misses/iter = **1.25**

```plaintext
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

**kij (& ikj):**
- 2 loads, 1 store
- misses/iter = **0.5**

```plaintext
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

**jki (& kji):**
- 2 loads, 1 store
- misses/iter = **2.0**

```plaintext
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```
Pentium Matrix Multiply Performance

- Miss rates are helpful but not perfect predictors.