Homework 4 Solution.

To show $L_1\alpha$ is regular, if $L$ is regular.

Look at all the final states that have an incoming transition labeled with "a."

Such states can be two forms.

\[ a \rightarrow \] change to.

\[ a \rightarrow \]

Then, make all other final states non-final (any other final states that do not have an incoming "a" transition.

To show if $L$ is regular $\Rightarrow$ all is also regular.

1. Let $D$ be the DFA for $L^R$.

2. Build a new DFA called $D_1\alpha$ for $L^R/\alpha$.

   $D_1\alpha \leftarrow$ DFA for language $L^R/\alpha$. (already proved in Q1)

3. Reverse that DFA

   \[ a \downarrow L = (L^R/\alpha)^R \]
\[ A = D = G \]
\[ B = H = E \]
\[ E = I = F \]
(a) \( L_1 = \{ a^i b^j c^k \mid i \neq j \}, \ i,j,k \geq 0 \):

\[
\begin{align*}
S_1 & \rightarrow Xa+b \ C \\
Xa+b & \Rightarrow a \ Xa+b \ b \ |A+|B+ \\
A+ & \Rightarrow aA+|a \\
B+ & \Rightarrow bB+|b \\
C & \Rightarrow cC|C \\
\end{align*}
\]

(b) \( L_2 = \{ a^i b^j c^k | i \neq k \}, \ i,j,k \geq 0 \):

\[
\begin{align*}
S_2 & \rightarrow a \ S_2 \ c \ |X|Y|1 \\
X & \Rightarrow B \ C+ \\
B & \Rightarrow bB|b \\
C & \Rightarrow cC|C \\
\end{align*}
\]

\[
y = 2A+13 \\
A+ = 2aA+|a \\
\]

(c) \( L_3 = \{ a^i b^j c^k \mid i \neq j \text{ or } i \neq k \} \):

\[
S_3 \rightarrow S_1 \ S_2 .
\]
a) \[ S \Rightarrow aSbS | bSaS | \lambda \]
A \Rightarrow \alpha A | \lambda

e) \[ S \Rightarrow aA1bA | a1b \]
On \[ S \Rightarrow aQS | aQS | bS | bS | bS | \lambda1b \]
A \Rightarrow aS | \lambda b

f) Set of strings \[ \text{www} \]
\[ S \Rightarrow aS1 | bS1 | aTB | bTA | a1b \]
T \Rightarrow aT | bT | \lambda

\[ \text{Leftmost} \quad \text{A | B} \]
\[ \text{0A | 1B} \]
\[ 00A | 1B \]
\[ 000A | 1B \]
\[ 0001 | B \]
\[ 0001 | B \]
\[ 0001 \]

\[ \text{Rightmost} \quad \text{A | B} \]
\[ \text{A | 1B} \]
\[ A | \lambda \]
\[ \text{0A | 1} \]
\[ 00A | 11 \]
\[ 000A | 11 \]
\[ 0001 | B \]
\[ 0001 | B \]
\[ 0001 \]