Homework 5
CptS 317, Spring 2019

Due Date: March 27, 2019

Total points: 48

1. (5 points)
Recall the following CFG G we discussed in the class:

E → E + T | T
T → T × F | F
F → (E) | a

Give parse trees for the following strings:

1. a × (a + a)
2. a + a + a
3. ((a + (a)))

2. (6 points) (from Exercise 5.1.7 in book): Consider

the CFG G defined by the productions:

S → 0S|S|0|1

Prove by induction that no string in L(G) has 10 as a substring. Hint: To show this, you will have to do induction on the length of the string, for strings that be generated by the grammar.

3. (12 points)

Give simple English language descriptions for the following grammars:

a) G_1 : S → 0S | 0
b) G_2 : S → 0S0 | 00|0

c) G_3 : S → S0S | 0

d) G_4 : S → 0S0 | 0S1 | 1S0 | 0
4. (5 points) Is the following grammar ambiguous? Justify your answer (give a string and derive it with two leftmost derivations using G). If it is ambiguous, rewrite this grammar to an unambiguous one (hint: recall “dangling else” as we discussed in the class).

\[
G : S \rightarrow 0S \mid 0S1S \mid \epsilon
\]

5. (5 points) Give context-free grammar that generate the following languages.
   (1) \{w is a binary string, and w starts and ends with the same symbol\};
   (2) the empty language (empty set).

6. (5 points) Recall right-linear CFGs: each production body has at most one variable (nonterminal symbol), and that variable is at the right end; in other words, all productions of a right-linear CFG are of form \(A \rightarrow wB\) or \(A \rightarrow B\), where \(A\) and \(B\) are variables (nonterminal symbols) and \(w\) is some string of zero or more terminal symbols. Give a right-linear CFG for the language of all binary strings that contain substring ‘00’.

7. (10 points)

Consider you want to program a robot to move in a 2D map. An instruction is a command from a set \{“up”, “down”, “left”, “right”\}. For example, instruction “up” tells the robot to move up from its current location (initialized as (0,0)); similarly for “down” (move down), “left” (move left), and “right” (move right). A sequence is a series of instructions that always starts with a keyword “begin”; for example, “begin left down right right right up up” gives a trajectory of the robot as in the right figure.

1. Give a CFG that can generate all sequences. You may use nonterminal symbols Seq (for sequence), Instr (for instructions), and the terminal symbols include ‘begin’ among others.

2. Consider the following maze. Use your grammar to generate a sequence that directs the
robot to the exit. (blue: where your robot is; green: the exit; dark blocks: walls). Consider a new maze obtained by removing wall blocks 1 and 4. Give another derivation that yields a shortest sequence that can direct the robot to exit in the new maze.