Homework 8
Cpt S 317, Spring 2019

Due Date: April 25, 2018 (Wednesday)

Total points: 38

For questions that ask you for a Turing Machine (TM) design in this homework:

Give your answer in the form of a state diagram. Your design should be for a basic form of
the TM (i.e., single tape, single track, no storage), following a similar design representation
as in TM’s lecture notes slide #8 and as was done in class.

1. (18 points) Let:

\[ L_1 = \{ a^n b^2 m c^m \mid m, n \geq 1 \} \]
\[ L_2 = \{ a^n b^m c^{2m} \mid m, n \geq 1 \} \]

a) Give CFGs for \( L_1 \) and \( L_2 \).
b) Is \( L_1 \cap L_2 \) a CFL? Justify your answer.
c) Using the CFG designed for \( L_1 \) as a template, design another CFG for the language
(denoted as \( L_{\text{pref}} \)) of all strings that are prefixes of the strings in \( L_1 \) — i.e.,

\[ L_{\text{pref}} = \{ x \mid x \text{ is a prefix of a string in } L_1 \} \]

Note, by this definition, each string in \( L_1 \) will generate multiple strings in \( L_{\text{pref}} \). For
example, the string \( abbcc \) which is in \( L_1 \) will generate the following list of strings in
\( L_{\text{pref}} \): \{\( \epsilon \), \( a \), \( ab \), \( abb \), \( abc \), \( abbcc \)\}.
2. (7 points) Design a Turing Machine for the language of strings of the form:
\[ a^n b^n c^m \]
where \( m \geq n \), and \( m, n \geq 0 \).

Answer the question either in the form of a state machine (preferred), or provide an English language step-by-step pseudocode describing the main logic of your TM design.

3. (7 points) Exercise 8.2.3 part a. Give the answer in the form of a state diagram.

4. (6 points) Exercise 8.2.5 part b. Give the answer in the form of a state diagram.