# Signaling P Systems and Verification Problems*

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**Abstract.** We introduce a new model of membrane computing system (or P system), called signaling P system. It turns out that signaling systems are a form of P systems with promoters that have been studied earlier in the literature. However, unlike non-cooperative P systems with promoters, which are known to be universal, non-cooperative signaling systems have decidable reachability properties. Our focus in this paper is on verification problems of signaling systems; i.e., algorithmic solutions to a verification query on whether a given signaling system satisfies some desired behavioral property. Such solutions not only help us understand the power of "maximal parallelism" in P systems but also would provide a way to validate a (signaling) P system in vitro through digital computers when the P system is intended to simulate living cells. We present decidable and undecidable properties of the model of non-cooperative signaling systems using proof techniques that we believe are new in the P system area. For the positive results, we use a form of "upper-closed sets" to serve as a symbolic representation for configuration sets of the system, and prove decidable symbolic model-checking properties about them using backward reachability analysis. For the negative results, we use a reduction via the undecidability of Hilbert's Tenth Problem. This is in contrast to previous proofs of universality in P systems where almost always the reduction is via matrix grammar with appearance checking or through Minsky's two-counter machines. Here, we employ a new tool using Diophantine equations, which facilitates elegant proofs of the undecidable results. With multiplication being easily implemented under maximal parallelism, we feel that our new technique is of interest in its own right and might find additional applications in P systems.

## 1 Introduction

P systems [18, 19] are abstracted from the way the living cells process chemical compounds in their compartmental structure. A P system consists of a finite number of

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membranes, each of which contains a multiset of objects (symbols). The membranes are organized as a Venn diagram or a tree structure where a membrane may contain other membranes. The dynamics of the P system is governed by a set of rules associated with each membrane. Each rule specifies how objects evolve and move into neighboring membranes. In particular, a key feature of the model of P systems is that rules are applied in a nondeterministic and maximally parallel manner. Due to the key feature inherent in the model, P systems have a great potential for implementing massively concurrent systems in an efficient way that would allow us to solve currently intractable problems (in much the same way as the promise of quantum and DNA computing). It turns out that P systems are a powerful model: even with only one membrane (i.e., 1-region P systems) and without priority rules, P systems are already universal [18, 22]. In such a one-membrane P system, rules are in the form of $u \to v$, which, in a maximally parallel manner, replaces multiset $u$ (in current configuration which is a multiset of symbol objects) with multiset $v$.

Signals are a key to initiate biochemical reactions between and inside living cells. Many examples can be found in a standard cell biology textbook [3]. For instance, in signal transduction, it is known that guanine-nucleotide binding proteins (G proteins) play a key role. A large heterotrimeric G protein, one of the two classes of G proteins, is a complex consisting of three subunits: $G_\alpha$, $G_\beta$, and $G_\gamma$. When a ligand binds to a G protein-linked receptor, it serves as a signal to activate the G protein. More precisely, the GDP, a guanine nucleotide, bound to the $G_\alpha$ subunit in the unactivated G protein is now displaced with GTP. In particular, the G protein becomes activated by being dissociated into a $G_\beta$-$G_\gamma$ complex and a $G_\alpha$-GTP complex. Again, the latter complex also serves as a signal by binding itself to the enzyme adenylyl cyclase. With this signal, the enzyme becomes active and converts ATP to cyclic AMP. As another example, apoptosis (i.e., suicide committed by cells, which is different from necrosis, which is the result from injury) is also controlled by death signals such as a CD95/Fas ligand. The signal activates caspase-8 that initiates the apoptosis. Within the scope of Natural Computing (which explores new models, ideas, paradigms from the way nature computes), motivated by these biological facts, it is a natural idea to study P systems, a molecular computing model, augmented with a signaling mechanism.

In this paper, we investigate one-membrane signaling P systems (signaling systems in short) where the rules are further equipped with signals. More precisely, in a signaling system $M$, we have two types of symbols: object symbols and signals. Each configuration is a pair consisting of a set $S$ of signals and a multiset $\alpha$ of objects. Each rule in $M$ is in the form of $s, u \to s', v$ or $s, u \to \Lambda$, where $s, s'$ are signals and $u, v$ are multisets of objects. The rule is enabled in the current configuration $(S, \alpha)$ if $s$ is present in the signal set $S$ and $u$ is a sub-multiset of the multiset $\alpha$. All the rules are fired in maximally parallel manner. In particular, in the configuration as a result of the maximally parallel move, the new signal set is formed by collecting the set of signals $s'$ that are emitted from all the rules actually fired during the move (and every signal in the old signal set disappears). Hence, a signal may trigger an unbounded number of rule instances in a maximally parallel move.

We focus on verification problems of signaling systems; i.e., algorithmic solutions to a verification query on whether a given signaling system does satisfy some desired behavioral property. Such solutions not only help us understand the power of the maximally parallelism that is pervasive in P systems but also would provide a way to validate a (signaling) P system in vitro through digital computers when the P system is intended to simulate living cells. However, since one-membrane P systems are Turing-complete, so are signaling systems. Therefore, to study the verification problems, we have to look at restricted signaling systems. A signaling system is non-cooperative if each rule is in the form of $s, a \to \Lambda$ or in the form of $s, a \to s', bc$, where $a, b, c$ are object symbols. All the results can be generalized to non-cooperative signaling systems augmented with rules $s, a \to s', v$. We study various reachability queries for non-cooperative signaling systems $M$; i.e., given two formulas $Init$ and $Goal$ that define two sets of configurations, are there configurations $C_{\text{init}}$ in $Init$ and $C_{\text{goal}}$ in $Goal$ such that $C_{\text{init}}$ can reach $C_{\text{goal}}$ in zero or more maximally parallel moves in $M$? We show that, when $Init$ is a Presburger formula (roughly, in which one can compare integer linear constraints over multiplicities of symbols against constants) and $Goal$ is a region formula (roughly, in which one can compare multiplicities of symbols against constants), the reachability query is decidable. Notice that, in this case, common reachability queries like halting and configuration reachability are expressible. We also show that introducing signals into P systems indeed increases its computing power; e.g., non-cooperative signaling systems are strictly stronger than non-cooperative P systems (without signals). On the other hand, when $Goal$ is a Presburger formula, the query becomes undecidable. Our results generalize to queries expressible in a subclass of a CTL temporal logic and to non-cooperative signaling systems with rules $S, a \to S', v$ (i.e., the rule is triggered with a set of signals in $S$). We also study the case when a signal has bounded strength and, in this case, non-cooperative signaling systems become universal.

Non-cooperative signaling systems are also interesting for theoretical investigation, since the signaling rules are context-sensitive and the systems are still nonuniversal as we show. In contrast to this, rules $a \to v$ in a non-cooperative P system are essentially context-free. It is difficult to identify a form of restricted context-sensitive rules that are still nonuniversal. For instance, a communicating P system (CPS) with only one membrane [21] is already universal, where rules are in the form of $ab \to a_x b_y$ or $ab \to a_x b_y c_{come}$ in which $a, b, c$ are objects (the *c comes* from the membrane's external environment), $x, y$ (which indicate the directions of movements of $a$ and $b$) can only be $here$ or $out$. Also one membrane catalytic systems with rules like $Ca \to Cv$ (where $C$ is a catalyst) are also universal. More examples including non-cooperative signaling systems with promoters, which will be discussed further in this section, are also universal. Our non-cooperative signaling systems use rules in the form of $s, a \to s', v$, which are in a form of context-sensitive rules, since the signals constitute part of the triggering condition as well as the outcome of the rules.

At the heart of our decidability proof, we use a form of upper-closed sets to serve as a symbolic representation for configuration sets and prove that the symbolic representation is invariant under the backward reachability relation of a non-cooperative signaling system. From the studies in symbolic model-checking [7] for classic transition systems, our symbolic representation also demonstrates a symbolic model-checking procedure at least for reachability. In our undecidability proofs, we use the well-known result on the Hilbert's Tenth Problem: any r.e. set (of integer tuples) is also Diophantine. We note that, for P systems that deal with symbol objects, proofs for universality almost always use the theoretical tool through matrix grammar with appearance checking [16] or through Minsky's two-counter machines. Here, we employ a new tool using Diophantine equations, which facilitates elegant proofs of the undecidable results. With multiplication being easily implemented under maximal parallelism, we feel that our new technique is of interest in its own right and might find additional applications in P systems.

Signaling mechanisms have also been noticed earlier in P system studies. For instance, in a one-membrane P system with promoters [4], a rule is in the form of $u \to v|p$ where $p$ is a multiset called a promoter. The rule fires as usual in a maximally parallel manner but only when objects in the promoter all appear in the current configuration. Notice that, since $p$ may not be even contained in $u$, a promoter, just as a signal, may trigger an unbounded number of rule instances. Indeed, one can show that a signaling system can be directly simulated by a one-membrane P system with promoters. However, since one-membrane non-cooperative P systems with promoters are known to be universal [4], our decidability results on non-cooperative signaling systems have a nice implication: our signals are strictly weaker than promoters (and hence have more decidable properties). The decidability results also imply that, as shown in the paper, non-cooperative signaling systems and vector addition systems (i.e., Petri nets) have incomparable computing power, though both models have a decidable configuration-to-configuration reachability. This latter implication indicates that the maximal parallelism in P systems and the "true concurrency" in Petri nets are different parallel mechanisms. Other signaling mechanisms such as in [2] are also promoter-based.

## 2 Preliminaries

We use $\mathbf{N}$ to denote the set of natural numbers (including 0) and use $\mathbf{Z}$ to denote the set of integers. Let $\Sigma = \{a_1, \cdots, a_k\}$ be an alphabet, for some $k$, and $\alpha$ be a (finite) multiset over the alphabet. In this paper, we do not distinguish between different representations of the multiset. That is, $\alpha$ can be treated as a vector in $\mathbf{N}^k$ (the components are the multiplicities of the symbols in $\Sigma$); $\alpha$ can be treated as a word on $\Sigma$ where we only care about the counts of symbols (i.e., its Parikh map). For a $\sigma \subseteq \Sigma$, we use $\sigma^*$ to denote the set of all multisets on $\sigma$.

A set $S \subseteq \mathbf{N}^k$ is a *linear set* if there exist vectors $v_0, v_1, \ldots, v_t$ in $\mathbf{N}^k$ such that $S = \{v \mid v = v_0 + a_1 v_1 + \cdots + a_t v_t,\ a_i \in \mathbf{N}\}$. A set $S \subseteq \mathbf{N}^k$ is *semilinear* if it is a finite union of linear sets. Let $x_1, \cdots, x_k$ be variables on $\mathbf{N}$. A *Presburger formula* is a Boolean combination of linear constraints in the following form: $\sum_{1 \le i \le k} t_i \cdot x_i \sim n$, where the $t_i$'s and $n$ are integers in $\mathbf{Z}$, and $\sim \in \{>, <, =, \ge, \le, \equiv_m\}$ with $0 \ne m \in \mathbf{N}$. It is known that a set of multisets (treated as vectors) is semilinear iff the set is definable by a Presburger formula. Also, Presburger formulas are closed under quantification.

A signaling system is simply a P system [18] augmented with signals. Formally, a (1-membrane) *signaling system* $M$ is specified by a tuple $\langle \Sigma, Sig, R \rangle$, where $\Sigma = \{a_1, \cdots, a_k\}$ is the alphabet, $Sig$ is a nonempty finite set of *signals*, and $R$ is a finite set of *rules*. Each rule is in the form of $s, u \to s', v$, where $s, s' \in Sig$ and $u$ and $v$ are multisets over alphabet $\Sigma$. (Notice that a rule like $s, u \to v$ (without emitting signal) can be treated as a short hand of $s, u \to s_{\text{garbage}}, v$ where $s_{\text{garbage}}$ is a "garbage" signal that won't trigger any rules.) A *configuration* $C$ is a pair consisting of a set $S$ of signals and a multiset $\alpha$ on $\Sigma$. As with the standard semantics of P systems [18, 19, 20], each evolution step, called a *maximally parallel move*, is a result of applying all the rules in $M$ in a maximally parallel manner. More precisely, let $s_i, u_i \to s'_i, v_i$, $1 \leq i \leq m$, be all the rules in $M$. We use $\mathbf{R} = (r_1, \cdots, r_m) \in \mathbf{N}^m$ to denote a multiset of rules, where there are $r_i$ instances of rule $s_i, u_i \to s'_i, v_i$, for each $1 \leq i \leq m$. Rule $s_i, u_i \to s'_i, v_i$ is *actually fired* in $\mathbf{R}$ if $r_i \geq 1$ (there is at least one instance of the rule in $\mathbf{R}$). Let $C = (S, \alpha)$ and $C' = (S', \alpha')$ be two configurations. The rule multiset $\mathbf{R}$ is *enabled* under configuration $C$ if

- multiset $\alpha$ contains multiset $\cup_{1 \leq i \leq m} r_i \cdot u_i$ (i.e., the latter multiset is the multiset union of $r_i$ copies of multiset $u_i$, for all $1 \leq i \leq m$), and
- set $S \supseteq \{s_i : r_i > 0, 1 \leq i \leq m\}$ (i.e., for every rule actually fired in $\mathbf{R}$, the signal $s_i$ that triggers the rule must appear in the set $S$ of the configuration $C$).

(We say that a rule is enabled under configuration $C$ if the rule multiset that contains exactly one instance of the rule is enabled under the configuration.) The result $C' = (S', \alpha')$ of applying $\mathbf{R}$ over $C = (S, \alpha)$ is as follows: set $S'$ is obtained by replacing the entire $S$ by the new signal set formed by collecting all the signals $s'_i$ emitted from the rules that are actually fired in $\mathbf{R}$, and, multiset $\alpha'$ is obtained by replacing, in parallel, each of the $r_i$ copies of $u_i$ in $\alpha$ with $v_i$. The rule multiset $\mathbf{R}$ is *maximally enabled* under configuration $C$ if it is enabled under $C$ and, for any other rule multiset $\mathbf{R}'$ that properly contains $\mathbf{R}$, $\mathbf{R}'$ is not enabled under the configuration. Notice that, for the same $C$, a maximally enabled rule multiset may not be unique (i.e., $M$ is in general nondeterministic). $C$ can reach $C'$ through a maximally parallel move, written $C \to_M C'$, if there is a maximally enabled rule multiset $\mathbf{R}$ such that $C'$ is the result of applying $\mathbf{R}$ over $C$. We use $C \leadsto_M C'$ to denote the fact that $C'$ is reachable from $C$; i.e., for some $n$ and $C_0, \cdots, C_n$, we have $C = C_0 \to_M \cdots \to_M C_n = C'$. We simply say that $C$ is reachable if the initial configuration $C'$ is understood. We say that configuration $C$ is *halting* if there is no rule enabled in $C$.

When the signals are ignored in a signaling system, we obtain a 1-membrane P system. Clearly, signaling systems are universal, since, as we have mentioned earlier, 1-membrane P systems are known to be universal. A non-cooperative signaling system is a signaling system where each rule is either a *split-rule* in the form of $s, a \to s', bc$ or a *die-rule* in the form of $s, a \to \Lambda$, where $s, s' \in Sig$ and symbols $a, b, c \in \Sigma$. The two rules are called $a$-rules (since $a$ appears at the LHS). Intuitively, the split-rule, when receiving signal $s$, makes an $a$-object split into a $b$-object and a $c$-object with signal $s'$ emitted. On the other hand, the die-rule, when receiving signal $s$, makes an $a$-object die (i.e., becomes null). In particular, for a configuration $C$, an $a$-object is *enabled* in $C$ if there is an enabled $a$-rule in $C$; in this case, we also call $a$ to be *an enabled symbol in $C$*. In the rest of the paper, we will focus on various reachability queries for non-cooperative signaling systems.

## 3 Configuration Reachability

We first investigate the *configuration-reachability* problem that decides whether one configuration can reach another.

**Given:** a non-cooperative signaling system $M$ and two configurations $C_{\rm init}$ and $C_{\rm goal}$,
**Question:** Can $C_{\rm init}$ reach $C_{\rm goal}$ in $M$?
In this section, we are going to show that the problem is decidable. The proof performs backward reachability analysis. That is, we first effectively compute (a symbolic representation of) the set of all configurations $C'$ such that $C' \rightsquigarrow_M C_{\rm goal}$. Then, we decide whether the initial configuration $C_{\rm init}$ is in the set.

Before proceeding further, we first introduce the symbolic representation. Let $\mathcal{C}$ be a set of configurations. We say that $\mathcal{C}$ is *upper-closed* if $\mathcal{C} = \{(S, \alpha) : \alpha$ is the multiset union of $\beta$ and some multiset in $\sigma^*\}$, for some $S \subseteq Sig$, multiset $\beta$ and some symbol-set $\sigma \subseteq \Sigma$. In this case, we use $[S, \beta, \sigma^*]$ to denote the set $\mathcal{C}$. We say that $\mathcal{C}$ is $m$-*bounded* if $|\beta| \leq m$. Let $\mathcal{C}$ be a finite union of upper-closed sets of configurations. The *pre-image* of $\mathcal{C}$ is defined as $Pre_M(\mathcal{C}) = \{C' : C' \rightarrow_M C \in \mathcal{C}\}$. We use $Pre^*_M(\mathcal{C})$ to denote the set of all configurations $C'$ such that $C' \rightsquigarrow C$ for some $C \in \mathcal{C}$. The main result of this section is as follows.

**Theorem 1.** *Let $\mathcal{C}$ be a finite union of upper-closed sets of configurations in $M$. Then, $Pre^*_M(\mathcal{C})$ can also be effectively represented as a finite union of upper-closed sets of configurations in $M$.*

The complex proof of Theorem 1 constructs an intermediate signaling system $\hat{M}$ whose $Pre^*_{\hat{M}}$ is easier to compute. The theorem can be established after we prove that $Pre^*_M$-computation can be realized by $Pre^*_{\hat{M}}$-computation and that $Pre^*_{\hat{M}}(\mathcal{C})$ can be effectively represented as a finite union of upper-closed sets.

Now, we can show that the configuration-reachability problem for non-cooperative signaling systems is decidable. This result implies that non-cooperative signaling systems are not universal (the set of reachable configurations is recursive). Notice that $\mathcal{C} = \{C_{\rm goal}\}$ is an upper-closed set. Since, from Theorem 1, $Pre^*_M(\mathcal{C})$ is effectively a finite union of upper-closed sets, one can also effectively answer the reachability at the beginning of this Section by checking whether $C_{\rm init}$ is an element in one of the upper-closed sets. Hence,

**Theorem 2.** *The configuration reachability problem for non-cooperative signaling systems is decidable.*

Reachability considered so far is only one form of important verification queries. In the rest of this section, we will focus on more general queries that are specified in the computation tree logic (CTL) [6] interpreted on an infinite state transition system [5]. To proceed further, more definitions are needed.

Let $M$ be a non-cooperative signaling system with symbols $\Sigma$ and signals $Sig$. We use variables $\#(a), a \in \Sigma$, to indicate the number of $a$-objects in a configuration and use variable $S$ over $2^{Sig}$ to indicate the signal set in the configuration. A *region formula* $F$ (the word "region" is borrowed from [1]) is a Boolean combination of formulas in the following forms: $\#(a) > n,\ \#(a) = n,\ \#(a) < n,\ S = sig$, where $a \in \Sigma$, $n \in \mathbf{N}$, and $sig \subseteq Sig$. Region-CTL formulas $f$ are defined using the following grammar:

$f ::= F \mid f \wedge f \mid f \vee f \mid \neg f \mid \exists \circ f \mid \forall \circ f \mid f\ \exists\mathcal{U}\ f \mid f\ \forall\mathcal{U}\ f$, where $F$ is a region formula. In particular, the eventuality operator $\exists \diamond f$ is the shorthand of $true\ \exists\mathcal{U}\ f$, and, its dual $\forall\Box f$ is simply $\neg \exists \diamond \neg f$. We use Region-CTL$^\diamond$ to denote a subset of the Region-CTL, where formulas are defined with: $f ::= F \mid f \wedge f \mid f \vee f \mid \neg f \mid \exists \circ f \mid \forall \circ f \mid \exists \diamond f \mid \forall \Box f$, where $F$ is a region formula. Each $f$ is interpreted as a set $[f]$ of configurations that satisfy $f$, as follows:

- $[F]$ is the set of configurations that satisfy the region formula $F$;
- $[f_1 \wedge f_2]$ is $[f_1] \cap [f_2]$; $[f_1 \vee f_2]$ is $[f_1] \cup [f_2]$; $[\neg f_1]$ is the complement of $[f_1]$;
- $[\exists \circ f_1]$ is the set of configurations $C_1$ such that, for some execution $C_1 \to_M C_2 \to_M \cdots$, we have $C_2 \in [f_1]$;
- $[\forall \circ f_1]$ is the set of configurations $C_1$ such that, for any execution $C_1 \to_M C_2 \to_M \cdots$, we have $C_2 \in [f_1]$;
- $[f_1\ \exists\mathcal{U}\ f_2]$ is the set of configurations $C_1$ such that, for some execution $C_1 \to_M C_2 \to_M \cdots$, we have $C_1, \cdots, C_n$ are all in $[f_1]$ and $C_{n+1}$ is in $[f_2]$, for some $n$;
- $[f_1\ \forall\mathcal{U}\ f_2]$ is the set of configurations $C_1$ such that, for any execution $C_1 \to_M C_2 \to_M \cdots$, we have $C_1, \cdots, C_n$ are all in $[f_1]$ and $C_{n+1}$ is in $[f_2]$, for some $n$.

Below, we use $P$ to denote a Boolean combination of Presburger formulas over the $\#(a)$'s and formulas in the form of $S = sig$, where $sig \subseteq Sig$. The Region-CTL model-checking problem for non-cooperative signaling systems is to answer the following question:

**Given**: a non-cooperative signaling system $M$, a Region-CTL formula $f$, and a Presburger formula $P$,

**Question**: Does every configuration satisfying $P$ also satisfy $f$?
It is known that the Region-CTL model-checking problem for non-cooperative P systems with rules $a \to b$ is undecidable [8]. From this result, one can show that the Region-CTL model-checking problem for non-cooperative signaling systems is undecidable as well.

**Theorem 3.** *The Region-CTL model-checking problem for non-cooperative signaling systems is undecidable.*

In contrast to Theorem 3, the subset, Region-CTL$^\diamond$, of Region-CTL is decidable for non-cooperative signaling systems:

**Theorem 4.** *The Region-CTL$^\diamond$ model-checking problem for non-cooperative signaling systems is decidable.*

Using Theorem 4, the following example property can be automatically verified for a non-cooperative signaling system $M$:

> "From every configuration satisfying $\#_a - \#_b < 6$, $M$ has some execution that first reaches a configuration with $\#_a > 15$ and then reaches a halting configuration containing the signal $s_1$ and with $\#_b < 16$."

Notice that, above, "halting configurations" (i.e., none of the objects is enabled) form a finite union of upper-closed sets.

## 4 Presburger Reachability

Let $M$ be a non-cooperative signaling system and $C_{\text{init}}$ be a given initial configuration. In this section, we are going to investigate a stronger form of reachability problems. As we have mentioned earlier, a multiset $\alpha$ (over alphabet $\Sigma$ with $k$ symbols) of objects can be represented as a vector in $\mathbf{N}^k$. Let $P(x_1, \cdots, x_k)$ be a Presburger formula over $k$ nonnegative integer variables $x_1, \cdots, x_k$. The multiset $\alpha$ *satisfies* $P$ if $P(\alpha)$ holds. A configuration $(S, \alpha)$ of the non-cooperative signaling system $M$ *satisfies* $P$ if $\alpha$ satisfies $P$. An *equality* is a Presburger formula in the form of $x_i = x_j$, for some $1 \leq i, j \leq k$. An *equality formula*, which is a special form of Presburger formulas, is a conjunction of a number of equalities. The *Presburger-reachability* problem is to decide whether a non-cooperative signaling system has a reachable configuration satisfying a given Presburger formula:

**Given:** a non-cooperative signaling system $M$, an initial configuration $C_{\text{init}}$, and a Presburger formula $P$,
**Question:** is there a reachable configuration satisfying $P$?
In contrast to Theorem 2, we can show that the Presburger-reachability problem is undecidable. The undecidability holds even when $M$ has only one signal (i.e., $|Sig| = 1$) and $P$ is an equality formula (i.e., the *equality-reachability problem*). In fact, what we will show is a more general result that characterizes the set of reachable configurations in $M$ satisfying $P$ exactly as r.e. sets. Notice that, for P systems that deal with symbol objects, proofs for universality almost always use the theoretical tool through matrix grammar with appearance checking [16]. Here, we employ a new tool using Diophantine equations. Before we proceed further, we recall some known results on Diophantine equations (the Hilbert's Tenth Problem).

Let $m \in \mathbf{N}$, $Q \subseteq \mathbf{N}^m$ be a set of natural number tuples, and $E(z_1, \cdots, z_m, y_1, \cdots, y_n)$ be a Diophantine equation system. The set $Q$ is *definable by* $E$ if $Q$ is exactly the solution set of $\exists y_1, \cdots, y_n. E(z_1, \cdots, z_m, y_1, \cdots, y_n)$; i.e., $Q = \{(z_1, \cdots, z_m) : E(z_1, \cdots, z_m, y_1, \cdots, y_n) \text{ holds for some } y_1, \cdots, y_n\}$. An *atomic* Diophantine equation is in one of the following three forms: $z = xy + \frac{1}{2}x(x+1), z = x + y, z = 1$, where $x, y, z$ are three distinct variables over $\mathbf{N}$. A conjunction of these atomic equations is called a Diophantine equation system of atomic Diophantine equations. It is well known that $Q$ is r.e. iff $Q$ is definable by some Diophantine equation system [17]. From here, it is not hard to show the following:

**Lemma 1.** *For any set $Q \subseteq \mathbf{N}^m$, $Q$ is r.e. iff $Q$ is definable by a Diophantine equation system of atomic Diophantine equations.*

We now build a relationship between Diophantine equations and non-cooperative signaling systems. Recall that $Q$ is a subset of $\mathbf{N}^m$. We say that $Q$ is $(M, C_{\text{init}}, P)$-*definable* if there are designated symbols $Z_1, \cdots, Z_m$ in $M$ such that, for any numbers $\#(Z_1), \cdots, \#(Z_m)$,

> $(\#(Z_1), \cdots, \#(Z_m))$ is in $Q$ iff there is a reachable configuration from $C_{\text{init}}$ in $M$ satisfying $P$ and, for each $i$, the number of $Z_i$-objects in the configuration is $\#(Z_i)$.

When $P$ is $true$ and $C_{\rm init}$ is understood, we simply say that $Q$ is definable by $M$. The non-cooperative signaling system $M$ is *lazy* if, for any reachable configuration and any number $n$, if the configuration is reachable from $C_{\rm init}$ in $n$ maximally parallel moves, then it is reachable in $t$ maximally parallel moves for any $t \geq n$. We first show that solutions to each atomic Diophantine equation can be defined with a lazy non-cooperative signaling system $M$ with only one signal.

**Lemma 2.** *The solution set to each atomic Diophantine equation is definable by some lazy non-cooperative signaling system $M$ (starting from some $C_{\rm init}$) with only one signal.*

Now, we can show the following characterization.

**Theorem 5.** *For any set $Q \subseteq \mathbf{N}^m$, $Q$ is r.e. iff $Q$ is $(M, C_{\rm init}, P)$-definable for some non-cooperative signaling system $M$ with one signal, some configuration $C_{\rm init}$, and some equality formula $P$.*

From Theorem 5, we immediately have

**Theorem 6.** *The equality-reachability problem for non-cooperative signaling systems with only one signal is undecidable. Therefore, the Presburger-reachability problem for non-cooperative signaling systems is undecidable as well.*

All the decidable/undecidable results presented so far can be generalized to the case when non-cooperative signaling systems are augmented with rules in the following forms: $s, a \to s', v$, where $v$ is a multiset. From now on, we let non-cooperative signaling systems contain these rules by default.

The results in Theorem 5 and Theorem 6 can be used to obtain a new result on non-cooperative P systems $\hat{M}$ where $\hat{M}$ has only one membrane and each rule is in the form of $a \to v$, where $v$ is a multiset. Notice that $\hat{M}$ is very similar to a non-cooperative signaling system $M$ with only one signal. Indeed, one can easily show that they are effectively equivalent in the following sense:

**Lemma 3.** *For any set $Q \subseteq \mathbf{N}^m$, $Q$ is definable by some non-cooperative P system $\hat{M}$ iff $Q$ is definable by some non-cooperative signaling system $M$ with only one signal.*

It is known that $\hat{M}$ is not a universal P system model; multisets generated from $\hat{M}$ form the Parikh map of an ET0L language [15]. We now augment $\hat{M}$ with a *Presburger tester* that, nondeterministically at some maximally parallel move during a run of $\hat{M}$, tests (*for only once*) whether the current multiset satisfies a given Presburger formula $P$. When $P$ is an equality formula, the tester is called an *equality tester*. If yes, the tester outputs the multiset and $\hat{M}$ shuts down. Otherwise, $\hat{M}$ crashes (with no output). Let $X_1, \cdots, X_m$ be designated symbols in $\hat{M}$. We say that $Q \subseteq \mathbf{N}^m$ is *output-definable* by $\hat{M}$ if $Q$ is exactly the set of tuples $(\#(X_1), \cdots, \#(X_m))$ in the output multisets. Directly from Lemma 3 and Theorem 5, one can show that non-cooperative P systems (as well as non-cooperative signaling systems with only one signal) with an equality tester are universal:

**Theorem 7.** *For any set $Q \subseteq \mathbf{N}^m$, $Q$ is r.e. iff $Q$ is the output-definable by a non-cooperative P system (as well as a non-cooperative signaling system with only one signal) with an equality (and hence Presburger) tester.*

Hence,

**Corollary 1.** *The equality-reachability problem for non-cooperative P systems is undecidable. Therefore, the Presburger-reachability problem is undecidable as well.*

With the current technology, it might be difficult to implement the equality tester device to achieve the universality, which requires, e.g., external multiset evaluation during an almost instantaneous chemical reaction process. As we already know, a more natural way to perform the evaluation is to wait until the system *halts*; i.e., none of the objects in the current configuration is enabled. In this way, one can similarly formulate the halting-definability and the Presburger/equality-halting-reachability problems for non-cooperative signaling systems as well as for non-cooperative P systems, which concern halting and reachable configurations (instead of reachable configurations). We first show that non-cooperative signaling systems with only one signal has semilinear halting-definable reachability sets. This result essentially tells us that the number of signals matters, as far as halting configurations are considered: non-cooperative signaling systems with multiple signals are strictly stronger than non-cooperative signaling systems with only one signal (as well as non-cooperative P systems). This is because a non-semilinear set like $\{(n, 2^n) : n \geq 0\}$ can be easily halting-definable by a non-cooperative signaling system.

**Theorem 8.** *For any $Q \subseteq \mathbf{N}^m$, $Q$ is a semilinear set iff $Q$ is halting-definable by a non-cooperative signaling system with only one signal (as well as by a non-cooperative P system).*

One can similarly augment $\hat{M}$ as well as $\hat{M}$ with a Presburger tester but only test and output when a halting configuration is reached; i.e., a *Presburger halting tester*. The following result shows that non-cooperative signaling systems with only one signal and with a Presburger halting tester are not universal, while non-cooperative signaling systems with two signals and with an equality halting tester are universal. That is, again, the number of signals matters.

**Theorem 9.** *For any $Q \subseteq \mathbf{N}^m$, (1). $Q$ is a semilinear set iff $Q$ is output-definable by a non-cooperative signaling system with only one signal (as well as a non-cooperative P system) and with a Presburger halting tester. (2). $Q$ is r.e. iff $Q$ is output-definable by a non-cooperative signaling system with two signals and with an equality (and hence Presburger) halting tester.*

From Theorem 9, we immediately have:

**Theorem 10.** *(1). The halting Presburger reachability problem for non-cooperative signaling systems with two signals is undecidable. (2). The halting Presburger reachability problem for non-cooperative signaling systems with only one signal is decidable.*

## 5 Discussions and Future Work

In our set-up, a signal in a non-cooperative signaling system $M$ has unbounded strength; i.e., it can trigger an unbounded number of instances of an enabled rule. If we restrict the strength of each signal in $M$ to be $B$ (where $B$ is a constant), the resulting $M$ is called a $B$-bounded non-cooperative signaling system. A move in such $M$ is still maximally parallel. However, each signal can fire at most $B$ instances of rules. From Theorem 2, we know that (unbounded) non-cooperative signaling systems are not universal. In contrast to this fact, we can show that bounded non-cooperative signaling systems are universal. The universality holds even when $B = 2$. The case for $B = 1$ is open.

There is an intimate relationship between some classes of P systems and VAS (vector addition systems, or, equivalently, Petri nets)[13, 14]. Though non-cooperative signaling systems as well as VAS are not universal, they are incomparable in terms of the computing power. This is because, the Presburger-reachability problem of VAS is decidable [9] while, as we have shown, the same problem for non-cooperative signaling systems is undecidable. On the other hand, the $Pre^*$-image of a non-cooperative signaling system is always upper-closed while this is not true for VAS.

In the definition of a non-cooperative signaling system, a rule is in the form of $s, a \to s', v$, where $s$ and $s'$ are signals. Now, we generalize the definition by allowing rules in the form of $S, a \to S', v$ where $S$ and $S'$ are sets of signals (instead of signals). The maximally parallel semantics of the rules can be defined similarly. The differences are that the rule is enabled when every signal in $S$ is in the current configuration and, after the rule is fired, every signal in $S'$ is emitted. Hence, the rule now is triggered by exactly all of the signals in $S$. Such a rule is called a *multi-signal rule*. Let $M$ be such a non-cooperative signaling system with multi-signal rules. The proof of Theorem 1 can be adapted easily for such an $M$. Therefore, Theorem 2 and Theorem 4 still hold for non-cooperative signaling system with multi-signal rules. In fact, the results can be further generalized as follows.

Our study of non-cooperative signaling system was restricted to one membrane. We can generalize the model to work on multiple membranes (as in the P system), where each membrane has a set of rules, and in each rule $S, a \to S', v$ (we are using multi-signal rules) we specify the "target" membranes where each object in $v$ as well as each signal in $S'$ are transported to. Notice that we do not use priority rules nor membrane dissolving rules. We call this generalized model as a multimembrane non-cooperative signaling system with multi-signal rules. Observe that multimembranes can be equivalently collapsed into one membrane through properly renaming (signal and object) symbols in a membrane. That is, each membrane is associated with a distinguished set of symbols. Of course, in doing so, the number of distinct symbols and signals in the reduced one-membrane system will increase as a function of the number of membranes in the original system. Therefore, Theorem 2 and Theorem 4 can be further generalized to multimembranes non-cooperative signaling systems with multi-signal rules.

It is known that there are nonuniversal P systems where the number of membranes induces an infinite hierarchy in terms of computing power [12]. However, the above generalization says that the hierarchy collapses for non-cooperative signaling systems. Is there a hierarchy in terms of the number of membranes for a restricted and nonuniversal form of signaling systems (which is stronger than non-cooperative signaling systems)? We might also ask whether for one-membrane signaling systems, there is a hierarchy in terms of the numbers of symbols and signals used (since the conversion described above from multimembrane to one membrane increases the number of symbols and signals). As defined, a non-cooperative signaling system is a "generator" of multisets. For a given configuration $C$, there may be many configurations $C'$ that satisfy $C \rightarrow_M C'$. Hence, a (maximally parallel) move is nondeterministic. Can we define an appropriate model of non-cooperative signaling system e.g., an "acceptor" of multisets (rather than a generator) such that the next move is unique (i.e., deterministic)? Deterministic P systems have been found to have some nice properties [11].

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